Synchronous and Induction Machines

Synchronous and Induction Machines
(EE-202)

by

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Overview

1. Syllabus

Module 1 - Alternators, Armature winding
Module 2 - Performance of an alternator
Module 3 - Salient pole machine, Parallel operation of alternators
Module 4 - Synchronous motor, Three phase induction motor
Module 5 - Circle diagrams, Starting & Braking of induction motors
Module 6 - Induction generator, Single-phase induction motor

Module 1

- Principle of Operation
- Construction
- Classification of Alternators
- Production of Sinusoidal Alternating emf
- Armature Windings
- Classification of Armature Windings
- Winding Factor
- EMF Equation
- Harmonics in Voltage Waveform

Module 2

- Armature Reaction
  - Armature Reaction at Unity Power Factor, Lagging Zero Power Factor and Leading Zero Power Factor
  - Armature Leakage Reactance, Armature Resistance, Synchronous Reactance and Synchronous Impedance
- Alternator on Load
  - Phasor Diagram
  - Load Characteristic of Alternator
- Experimental Determination
  - Effective Resistance of Armature
  - Open Circuit Test
  - Short Circuit Test
- Voltage Regulation
  - Synchronous Impedance Method or EMF Method
  - Ampere-Turn Method or MMF Method
  - Zero Power Factor or Potier Method
  - ASA(American Standards Association) Method

- Theory of Salient Pole Machine
  - Blondels Two Reaction Theory
  - Slip test (Determination of Xd & Xq)
- Parallel Operation of Alternators
  - Requirements for Parallel Operation
  - Methods of Synchronisation - Dark Lamp Method, Bright Lamp Method, Synchroscope
  - Effects of Changing Excitation
  - Load Sharing between Two Alternators

- Synchronous motor
  - Construction
  - Principle of Operation
  - Methods of Starting
  - Effects of Excitation
  - V-curve
  - Inverted V-curve
  - Equivalent Circuit
  - Phasor Diagram
  - Losses and Efficiency
- Three phase Induction Motor
  - Construction
  - Types
  - Principle of Operation
  - Torque-Slip Characteristics
  - Equivalent Circuit of Induction Motor
  - Phasor Diagram

Module 5

- Induction motor
  - Tests on Induction Motor
  - Circle diagram
  - Cogging, Crawling & Noise production
- Double Cage Induction Motor
  - Torque-Speed Curve
  - Equivalent Circuit
- Starting of Induction Motor
  - DOL Starter, Autotransformer Starter, Star-Delta Starter & Rotor Resistance Starter
- Braking of Induction Motors - Plugging, Dynamic Braking & Regenerative Braking
- Speed Control of Induction Motor - Stator Voltage Control, V/f Control & Rotor Resistance Control

Module 6

- Induction Generator
  - Principle of Operation
  - Grid Connected & Self-excited Operation
  - Comparison of Induction Generator with Alternator
- Synchronous Induction Motor
- Single-phase Induction Motor
  - Double Revolving Field Theory
  - Torque-Slip Curve
  - Equivalent Circuit
  - Types
    - Split-Phase Induction Motor
    - Capacitor-Start Induction Motor
    - Capacitor-Start and Capacitor-Run Induction Motor
    - Shaded Pole Induction Motor

2. Expected Outcome

After the successful completion of this course, the students will be able to

1. Identify alternator types, and appreciate their performance
2. Determine the voltage regulation and analyse the performance of alternators
3. Describe the principle of operation of synchronous motor and different applications.
4. Describe the principle of operation of 3-phase induction motors and select appropriate motor types for different applications.
5. Analyse the performance of 3-phase induction motors
6. Familiarize with principle of operation and application of 1 -phase induction motors.

3. Books

Text Book

1. Bimbra P. S., Electrical Machinery, 7/e, Khanna Publishers, 2011.
2. Nagrath J. and D. P. Kothari, Theory of AC Machines, Tata McGraw Hill, 2006. Reference Book

Reference Books

1. Say M. G., The Performance and Design of A. C. Machines, C B S Publishers, New Delhi, 2002.
2. Fitzgerald A. E., C. Kingsley and S. Umans, Electric Machinery, 6/e, McGraw Hill, 2003.
3. Langsdorf M. N., Theory of Alternating Current Machinery, Tata McGraw Hill, 2001.
4. Deshpande M. V., Electrical Machines, Prentice Hall India, New Delhi, 2011.
5. Charles I. Hubert, Electric Machines, Pearson, New Delhi 2007
6. Theodore Wilde, Electrical Machines, Drives and Power System, Pearson Ed. Asia 2001

Thank You

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Synchronous machine - AC machine in which rotor rotates at a speed which bears a constant relationship to the frequency of currents in the armature winding.

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Principle - Faraday's Laws of Electromagnetic Induction
- No need for commutator

2. Construction

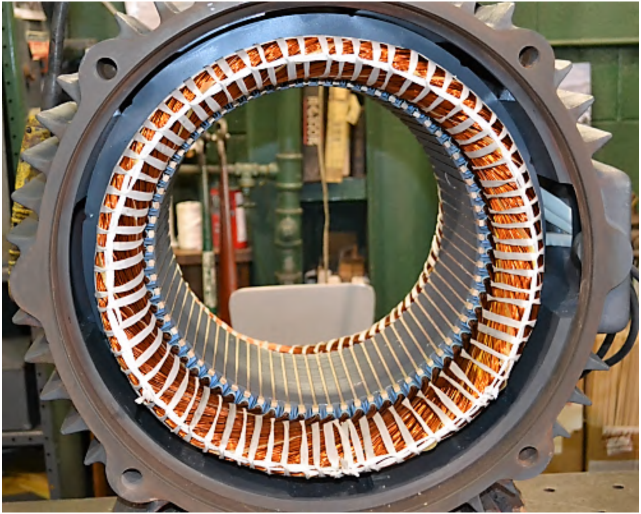

Figure 1 : Stator of Alternator

2. Construction

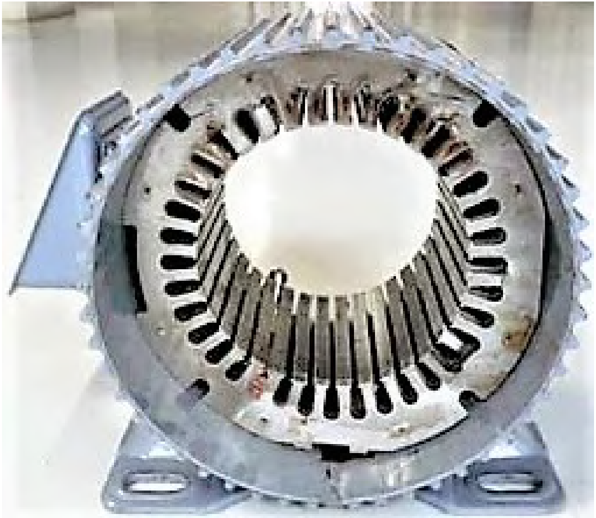

Figure 2 : Stator of Alternator

2. Construction

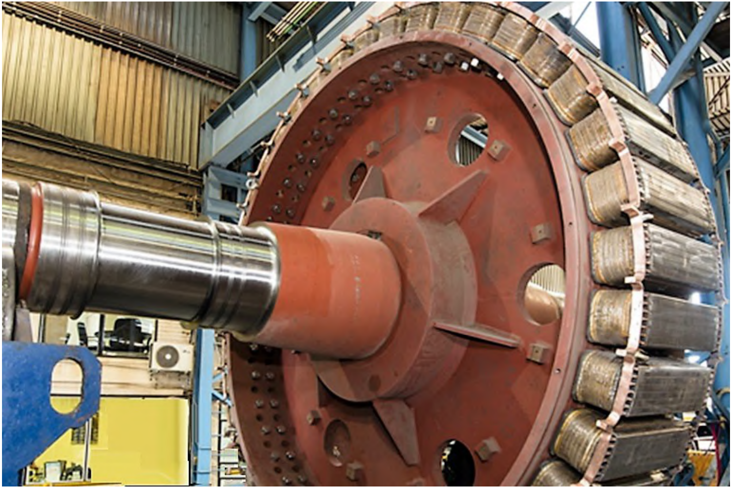

Figure 3 : Rotor of Alternator

2. Construction

Figure 4 : Rotor of Alternator

Armature System and Field System

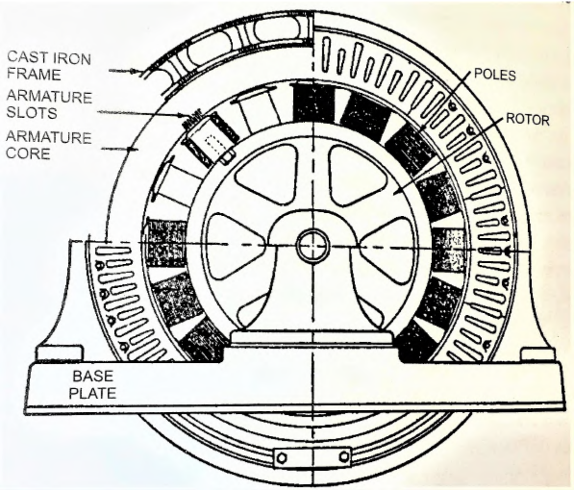

Figure 5 : Alternator

Armature System and **Field System**

A. Armature System

Armature Core - Laminated Silicon Steel sheets

Armature System and **Field System**

A. Armature System

Armature Core - Laminated Silicon Steel sheets

- Laminations reduce eddy current loss

2. Construction

Armature System and **Field System**

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Frame - Cast Iron

- Holds armature stampings

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- 3 types

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2. Semi-closed slot
3. Totally closed slot

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- 3 types

1. Open Slot
2. Semi-closed slot
3. Totally closed slot

- *Slot depth* is usually 4 to 6 times slot width
- *No. of slots per pole per phase(SPP)*

- For small machine : SPP is usually 3 or 4
- For large machine : SPP is usually greater than 5

B. Field System

- Field magnets

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  - Excited by current(Electromagnets) - Exciting current is supplied to rotor through two slip rings and brushes
  - Permanent magnets
- Alternate North(N) and South(S) poles
- Salient Pole and Non-salient Pole(Cylindrical) Rotor

(i) Salient Pole Rotor

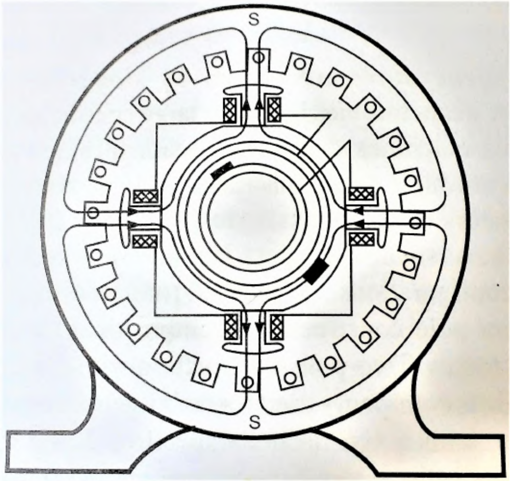

Figure 6 : Salient Pole Alternator

(i) Salient Pole Rotor

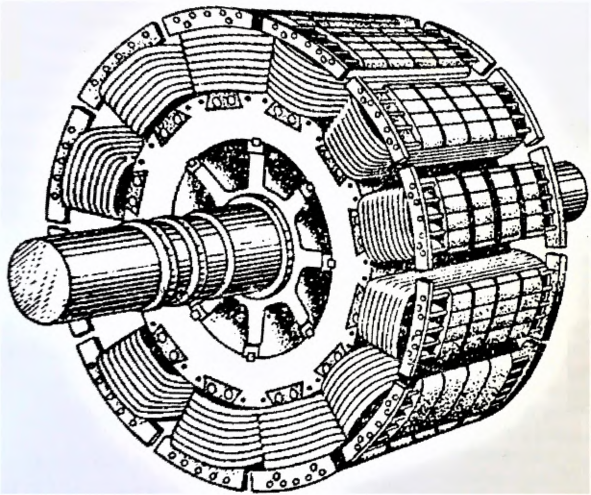

Figure 7 : Salient Pole Rotor

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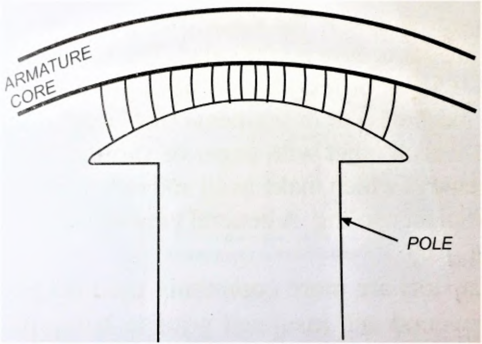

Figure 8 : Salient Pole

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- Pole shoe cover about 2/3rd of pole pitch

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- Large diameter and short axial length
- Pole shoe cover about 2/3rd of pole pitch
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Features

- Low speed machine
- Large diameter and short axial length
- Pole shoe cover about 2/3rd of pole pitch
- Poles are laminated to reduce eddy current loss
- Hydraulic Turbines(Low speed)

(ii) Non-Salient Pole Rotor or Cylindrical Type Rotor

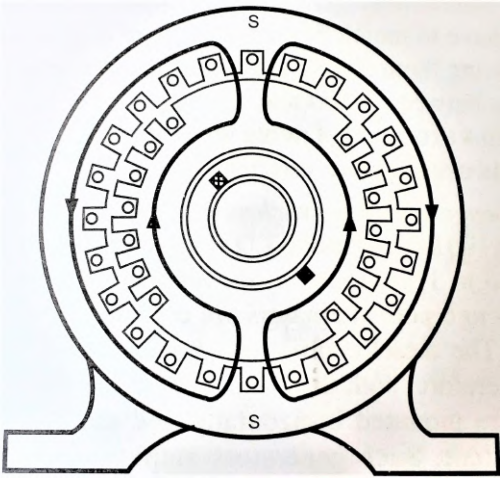

Figure 9 : Non-Salient Pole Alternator

(ii) Non-Salient Pole Rotor or Cylindrical Type Rotor

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Features

- High speed machine
- Small diameter and large axial length
- Less windage loss(air resistance)
- Nearly sinusoidal flux distribution around periphery $\implies$ better voltage waveform

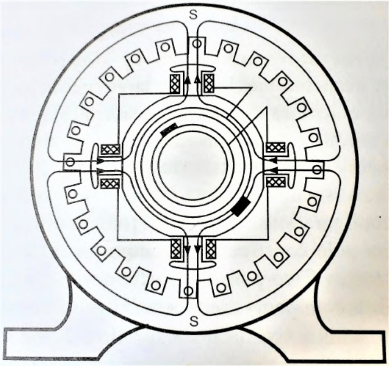

Figure 10 : Salient Pole Rotor

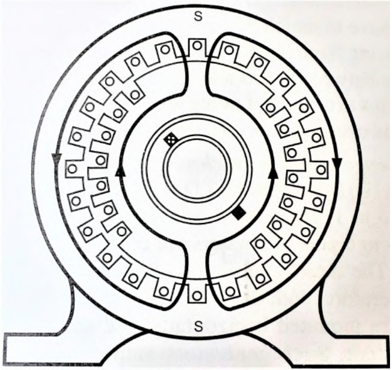

Figure 11 : Non-Salient Pole Rotor

3. Classification of Alternators

1. Based on **Application**
2. Based on **Output**
3. Based on **Construction**
4. Based on **Rotor Construction**
5. Based on **Type of Prime-mover**

A. Based on Application

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1. Synchronous Generator(Alternator)

3. Classification of Alternators

A. Based on Application

1. Synchronous Generator(Alternator)
2. Synchronous Motor

A. Based on Application

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   - Used to control the reactive power

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3. Synchronous Compensator
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   - Used to control the reactive power

B. Based on Output

1. Single phase alternator
2. Two phase alternator
3. Three phase alternator
   - Most commonly used

C. Based on Construction

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1. **Rotating Armature Type**

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1. **Rotating Armature Type**
   - Similar to DC generator **except that the commutator is replaced by 3 slip rings**
   - Field is produced by electromagnets placed in the stator
   - Current generated is collected by means of brushes and slip rings (from rotor)
   - Suitable for small low voltage generator**(**$<$250kVA**)**

2. Rotating Magnetic Field Type
   $\Longrightarrow$ - **Stationary Armature**

Advantages of Stationary Armature and Rotating field system

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- Load circuit can be directly connected with the fixed terminals of stator without passing through brushes and slip rings.

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  - Insulation of stationary armature winding is not subjected to any mechanical stresses.
  - More space will be available for providing more insulation as stator is outside the rotor.
- Load circuit can be directly connected with the fixed terminals of stator without passing through brushes and slip rings.
- Armature winding can be securely fixed on a rigid frame to prevent any deformation which could be developed by mechanical stresses.

Advantages of Stationary Armature and Rotating field system

- Armature winding is **cooled more readily**

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- Armature winding is **cooled more readily**
- Only 2 slip rings are required for the supply of dc to rotor(Low voltage - 125V or 250V).
- Due to simple and robust construction of rotor, higher speed of rotating magnetic field is possible

  $\Longrightarrow$ Increased output from the machine of given dimensions.

D. Based on Rotor Construction

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1. **Salient Pole Alternator**

   - Low speed machine

D. Based on Rotor Construction

1. **Salient Pole Alternator**
   - Low speed machine
2. **Non-salient Alternator**
   - Cylindrical rotor
   - High speed machine

E. Based on the Type of Prime-mover

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1. **Turbo generator**
   - Driven by steam turbine
   - High speed machine (3000rpm)
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   - Ratings upto 1000MVA

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   - Prime-mover is water turbine
   - Speed is between 100rpm and 1500rpm
   - Rating - upto 750MVA

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   - Cylindrical rotor
   - Ratings upto 1000MVA
2. **Hydro generator**
   - Prime-mover is water turbine
   - Speed is between 100rpm and 1500rpm
   - Rating - upto 750MVA
3. **Engine driven generator**
   - Prime-mover is usually diesel engine
   - Speed is between 375rpm to 1500rpm - Rating - upto 1000kVA

4. Production of Sinusoidal Alternating emf

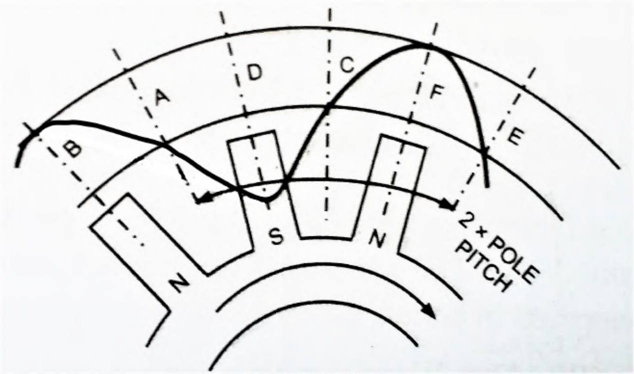

Figure 12 : Production of Alternating emf

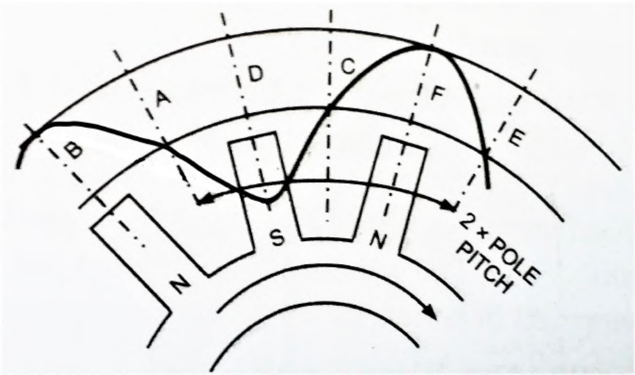

Figure 12 : Production of Alternating emf

- When the rotor is rotated, the armature conductors cut the magnetic flux and hence an emf is induced due to electromagnetic induction

- When conductor is opposite to neutral plane**(points A,C,E)** $\Longrightarrow$ **Zero emf** is induced

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- **Alternating emf completes one cycle in an angular distance equal to twice of pole pitch**.

- If the no. of poles on rotor is **P**, then

$\left(\frac{P}{2}\right)$ **cycles of emf are completed in one revolution**

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$$\implies f = \left(\frac{P}{2}\right)\left(\frac{N_s}{60}\right) = \frac{N_s P}{120} \text{ Hertz}$$

therefore,

- If the no. of poles on rotor is **P**, then

$\left(\frac{P}{2}\right)$ **cycles of emf are completed in one revolution**

Frequency = No. of cycles per second

$$\implies f = \left(\frac{P}{2}\right)\left(\frac{N_s}{60}\right) = \frac{N_s P}{120} \text{ Hertz}$$

therefore,

Synchronous speed $N_s = \dfrac{120f}{P}$ rpm

Armature Winding - An arrangement of conductors to develop desired emf by relative motion between the conductors and magnetic field

5. Armature Windings

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- Group of conductors placed in different slots
- Either Closed**(Delta connections)** or Open**(Star Connection)**

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General Guidelines

1. Span of each coil must be equal to pole pitch $\Longrightarrow$ Two sides of any coil must be under adjacent poles

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2. Coils must be connected such that emfs induced in them help each other

5. Armature Windings

Armature Winding - An arrangement of conductors to develop desired emf by relative motion between the conductors and magnetic field
- Group of conductors placed in different slots
- Either Closed**(Delta connections)** or Open**(Star Connection)**

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   - (Double Layer $\rightarrow$ Two coil sides per slot)
4. Winding must be designed such that it will approximately give induced emf of sinusoidal form

Terminology

- **Conductor :** Each individual length of wire lying within the magnetic field. Symbol used is **Z**
- **Turn :** When two conductors lying in magnetic field are connected in series, so that emf induced in them help each other, then it is called as a turn.
- **Coil :** One or more turns connected in series. Symbol is **C**

- **Coil Side :** Two sides of a coil embedded in two different slots nearly a pole pitch apart
- **Coil Group :** May have one or more single coils
- **Winding :** Number of coils arranged in coil groups
- **Pole Pitch :** Number of conductors per slot
- **Front Pitch(Y_f):** Distance in terms of no. of armature conductors between the second conductor of one coil and the first conductor of next coil
- **Back Pitch(Y_b) :** Distance in terms of no. of armature conductors between last and the first conductor of a coil.
- **Resultant Pitch(Y_r):** Distance in terms of no. of armature conductors between start of one coil to the start of next coil.
- **Coil Span/Coil Pitch :** When coil span is equal to pole pitch, $\implies$ *Full Pitched Coil*

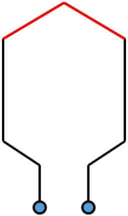

Figure 13 : Turn

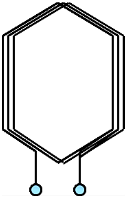

Figure 14 : Coil

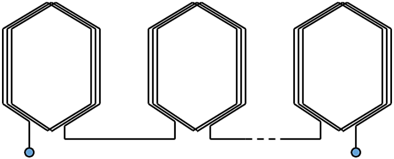

Figure 15 : Winding

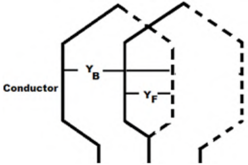

Figure 16 : Y_b, Y_f

Types -

1. **Single Phase Winding** and **Poly Phase Winding**

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2. **Concentrated Winding** and **Distributed Winding**

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6. Classification of Armature Windings

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7. **Integral Slot Winding** and **Fractional Slot Winding**

- Two Types

1. Concentrated Winding

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1. **Concentrated Winding** $\rightarrow$ No. of Slots = No. of Poles

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- Simplest possible winding is

a. Skelton Wave Winding

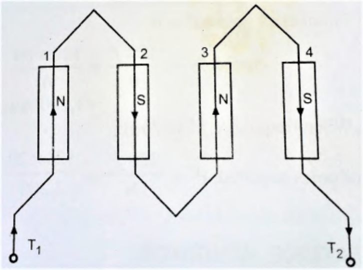

Figure 17 : Skelton Wave Winding

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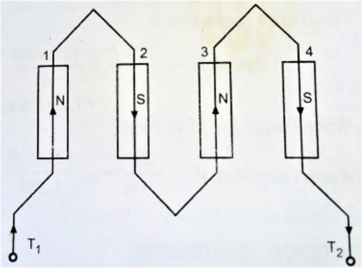

Figure 17 : Skelton Wave Winding

- **No. of conductors or coil sides is equal to no. of poles**

b. Half Coiled (or Hemi-tropic) Winding

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- Single turn coils of skelton wave winding are replaced with **multi-turn coils**

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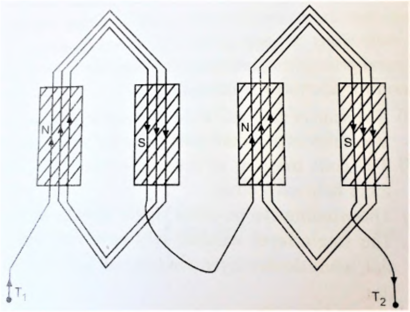

Figure 18 : Half Coiled or Hemi-tropic Winding

b. Half Coiled (or Hemi-tropic) Winding

- Single turn coils of skelton wave winding are replaced with **multi-turn coils**

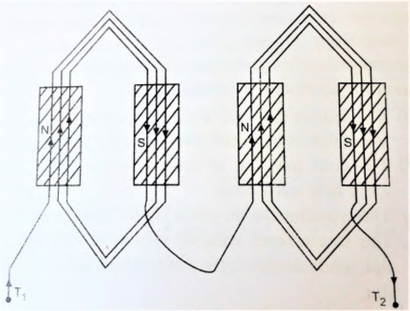

Figure 18 : Half Coiled or Hemi-tropic Winding

- Coils cover only half of the armature periphery → Hence the name **Half coiled or hemitropic winding**

c. Whole Coiled Winding

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- Coils are distributed over whole of the armature periphery → Hence the name **Whole Coiled Winding**
- No. of coils = no. of poles
- Coil side lying on top → Solid Line
- Coil side lying on bottom → Dotted Line → Double Layer Winding

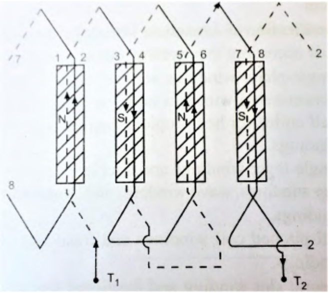

Figure 19 : Whole Coiled Winding

Single Phase Armature Windings

2. Distributed Winding

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Conductors are placed in several slots under one pole

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$\implies$ No. of Slots $\neq$ No. of Poles

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Advantages

- Harmonic emfs are reduced $\rightarrow$ More sinusoidal voltage waveform
- Diminishes armature reaction and armature reactance
- Effective Cooling(Even distribution of copper over the surface of armature)
- Core is better utilized as a number of small slots evenly spaced are employed

2. Distributed Winding

Types

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Types

a. Lap Winding

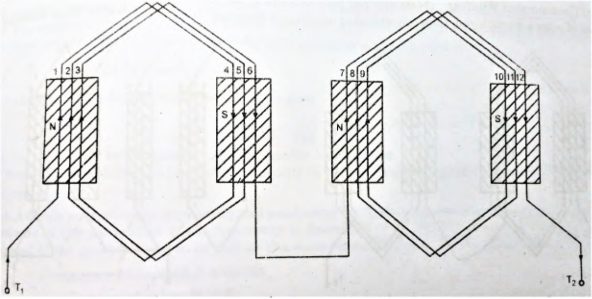

Figure 20 : 4-Pole 12-Slot 12-Conductor Single Layer Lap Winding

2. Distributed Winding

Types

a. Lap Winding

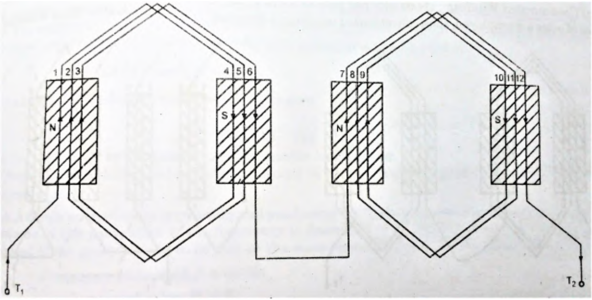

Figure 20 : 4-Pole 12-Slot 12-Conductor Single Layer Lap Winding

Back Pitch$(Y_b) = 3$, **Front Pitch**$(Y_f) = 2$

2. Distributed Winding

Types

a. Lap Winding

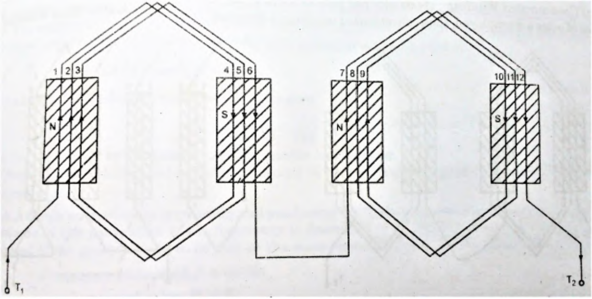

Figure 20 : 4-Pole 12-Slot 12-Conductor Single Layer Lap Winding

Back Pitch(Y_b) $= 3$, **Front Pitch**(Y_f) $= 2$

- Winding is completed per pair of pole and then connected in series

2. Distributed Winding

b. Wave Winding

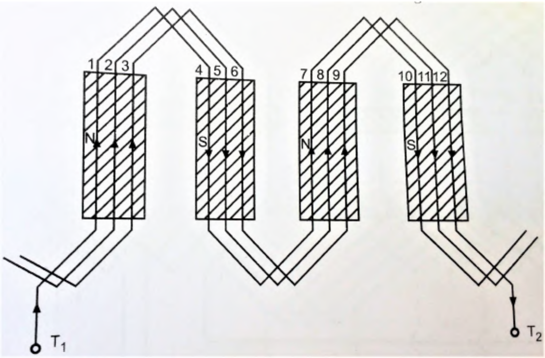

Figure 21 : 4-Pole, 12-Slot, 12-Conductor Single Layer Wave Winding

2. Distributed Winding

b. Wave Winding

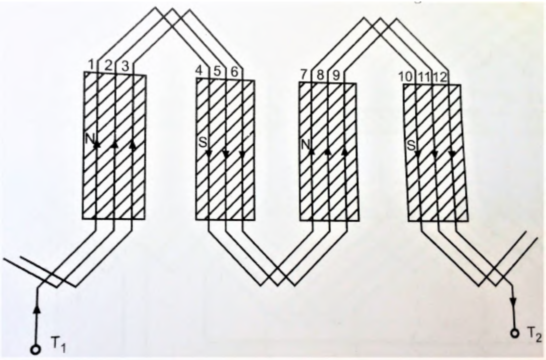

Figure 21 : 4-Pole, 12-Slot, 12-Conductor Single Layer Wave Winding

Back Pitch(Y_b) $= 3$, **Front Pitch**(Y_f) $= 3$

2. Distributed Winding
c. Concentric or Spiral Winding

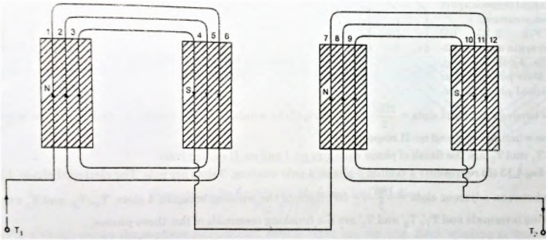

Figure 22 : 4-Pole, 12-Slot, 12-Conductor Single Layer Spiral Winding

2. Distributed Winding

c. Concentric or Spiral Winding

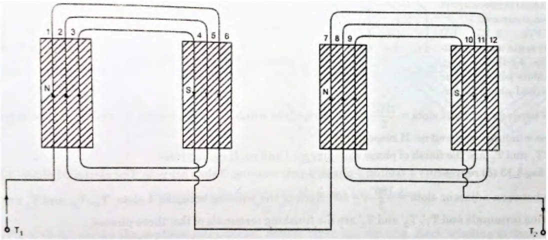

Figure 22 : 4-Pole, 12-Slot, 12-Conductor Single Layer Spiral Winding

- Coils have different pitches (Outer coil pitch=5, middle coil pitch=3, inner coil pitch=1)

Terms commonly used

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Poly-phase Armature Windings

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Poly-phase Armature Windings

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Three phase Distributed Winding

- Three separate windings placed 120^o apart

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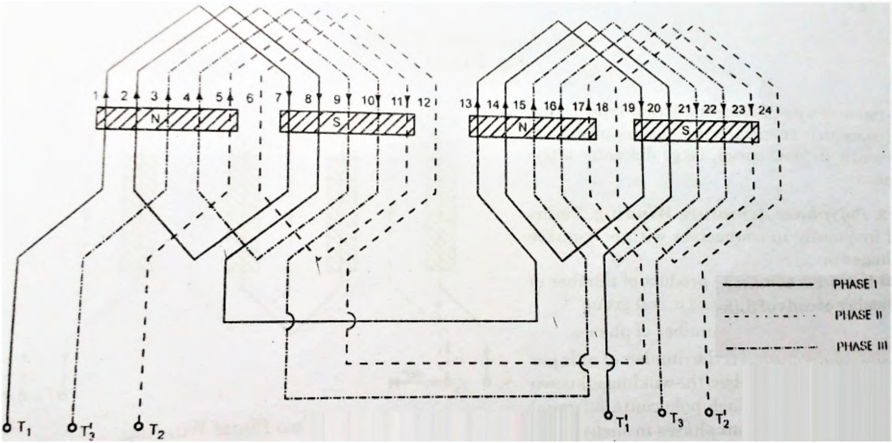

Figure 23 : 4-Pole, 24-Slot, Single Layer, 3-Phase Distributed Winding

Three phase Distributed Winding

* No. of slots per pole per phase $= \frac{24}{4 \times 3} = 2$

	Slot No
Phase I	1,2,7,8,13,14,19,20
Phase II	5,6,11,12,17,18,23,24
Phase III	3,4,9,10,15,16,21,22

Full Pitch Winding

- When two coil sides forming a complete coil of a winding are 180 electrical degree apart(**Coil span=**180^o), the winding is known as full pitched winding

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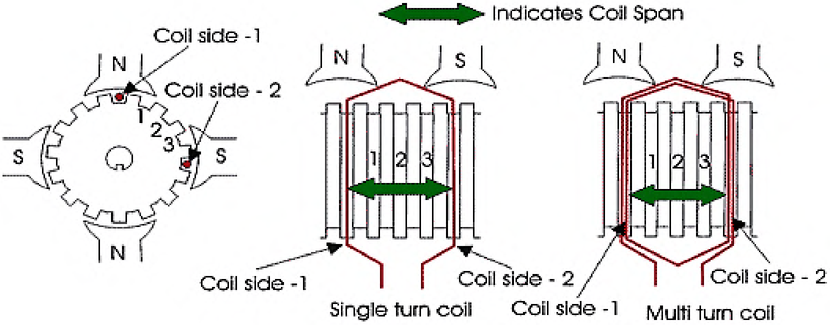
- Induced emfs differ by 180^o in phase but coils are connected such that emfs add each other giving higher resultant emf

Full Pitch Winding



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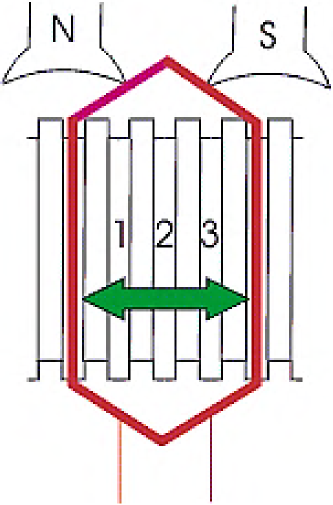

Figure 24 : Full Pitched Coil

Fractional or Short Pitch Winding

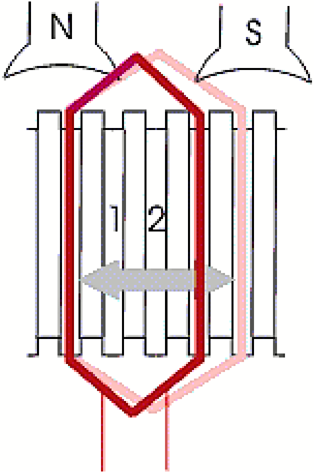

Figure 25 : Short Pitched Coil

Fractional or Short Pitch Winding

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- **Induced emf in short pitch winding** $<$ **emf induced in full pitch winding under same conditions**

Advantages of Short Pitch Winding

- More sinusoidal voltage waveform is generated
- Distorting harmonics can be reduced or totally eliminated
- Less amount of copper is required for end connection due to less coil span

Integral and Fractional Slot Windings

- No. of slots per pole per phase(SPP) = an integer

Integral and Fractional Slot Windings

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- No. of slots per pole per phase(SPP) $\neq$ an integer $\implies$ **Fractional Slot Winding**
- In both cases, no. of slot per phase must be an integer so that each phase has the same no. of coils

7. Winding Factor(K_w)

Winding Factor(K_w) = Pitch Factor(K_p) $\times$ Distribution Factor(K_d)

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Pitch Factor(K_p)

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Let

- coil have a pitch short by an angle of α electrical degree from full pitch
- Induced emf in each coil side be 'E'

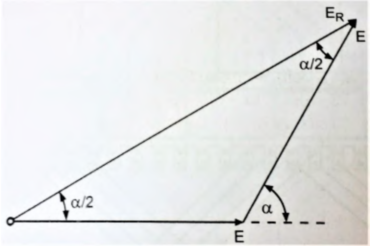

Figure 26 : Resultant Induced emf

- **Pitch Factor(K_p)/Coil Pitch Factor/Coil Span Factor** = Ratio of Phasor sum of coil side emfs and Arithmetic sum of coil side emfs

$$K_p = \frac{Phasor\ sum\ of\ coil\ side\ emfs}{Arithmetic\ sum\ of\ coil\ side\ emfs}$$

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- $\Longrightarrow$

$$K_p = \frac{2\ E\ Cos\left(\frac{\alpha}{2}\right)}{2\ E} = Cos\left(\frac{\alpha}{2}\right)$$

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- $\implies$

$$K_p = \frac{2\ E\ Cos\left(\frac{\alpha}{2}\right)}{2\ E} = Cos\left(\frac{\alpha}{2}\right)$$

- **Pitch Factor(K_p) is always less than unity for short pitched winding**

Pitch Factor(K_p)

- If the coil span is **reduced by one slot**, then phase angle(α) between the induced emfs in the two sides of the coil is given as

$$\alpha = \frac{180^o}{n}$$

where n = no. of slots per pole

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- r^{th} harmonic component of the flux wave, may be imagined as produced by **'r'** number of poles as compared to one pole for the fundamental component.

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- Chording angle for the r^{th} harmonic = **r** times the chording angle for the fundamental component
- Pitch Factor for the r^{th} harmonic

$$K_{pr} = Cos\left(\frac{r\,\alpha}{2}\right)$$

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- Pitch Factor for the r^{th} harmonic

$$K_{pr} = Cos\left(\frac{r\,\alpha}{2}\right)$$

- r^{th} harmonic emf can be reduced to zero if chording angle(α) is made such that

$$Cos\left(\frac{r\,\alpha}{2}\right) = 0 \implies \frac{r\,\alpha}{2} = 90^o$$

- r^{th} harmonic component of the flux wave, may be imagined as produced by **'r'** number of poles as compared to one pole for the fundamental component.
- Chording angle for the r^{th} harmonic = **r** times the chording angle for the fundamental component
- Pitch Factor for the r^{th} harmonic

$$K_{pr} = Cos\left(\frac{r\ \alpha}{2}\right)$$

- r^{th} harmonic emf can be reduced to zero if chording angle(α) is made such that

$$Cos\left(\frac{r\ \alpha}{2}\right) = 0 \implies \frac{r\ \alpha}{2} = 90^o$$

- If $\alpha = 30^o$,

Pitch Factor(K_p)

- r^{th} harmonic component of the flux wave, may be imagined as produced by **'r'** number of poles as compared to one pole for the fundamental component.
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- If $\alpha = 30^o$, Coil pitch $= 150^o$ electrical $\implies K_p = 0.966$, $K_{p3} = 0.707$ and $K_{p5} = 0.259$

- **Distributed winding** = Conductors are placed in several slots under one pole

Distribution Factor(K_d)

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- In distributed winding, coils per phase are displaced from each other by a certain angle
  $\Longrightarrow$ emfs induced in the coil sides of one phase in adjacent slots under one pole are not in phase with each other

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- **Distribution Factor(K_d)** or **Breadth Factor(K_b)** $=$ Ratio of phasor sum of emfs induced in all the coils distributed in a number of slots under one pole to the arithmetic sum of the emfs induced

Distribution Factor(K_d)

$$K_d = \frac{emf\ induced\ in\ distributed\ winding}{emf\ induced\ if\ the\ winding\ would\ have\ been\ concentrated}$$

$$K_d = \frac{Phasor\ sum\ of\ component\ emfs}{Arithmetic\ sum\ of\ component\ emfs}$$

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- **Distribution Factor(K_d) is always less than unity for distributed winding**

- Let n = No. of slots per pole
- m = No. of slots per pole per phase
- E_c = Induced emf in each coil side
- Angular displacement between the slots = β
- emf induced in different coils of one phase under one pole → AC, CD, EF,... → equal in magnitude(E_c) but differ in phase(β^o) from each other

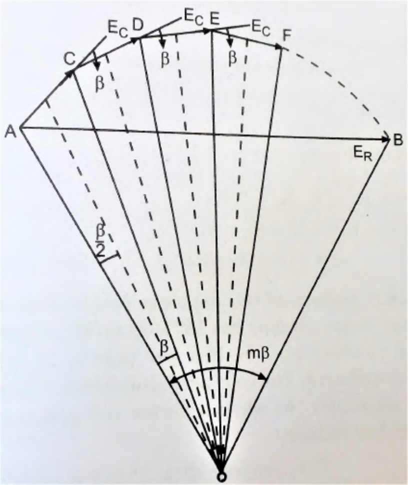

Figure 27 : Induced emf in coil sides in distributed winding

Distribution Factor(K_d)

- emf induced in each coil side,

$$E = AC = 2 \times OA \times Sin\left(\frac{\beta}{2}\right)$$

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- Resultant emf

$$E_R = AB = 2 \times OA \times Sin\left(\frac{m\beta}{2}\right)$$

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- **Distribution Factor(K_d)**

$$K_d = \frac{2 \times OA \times Sin\left(\frac{m\beta}{2}\right)}{m \times 2 \times OA \times Sin\left(\frac{\beta}{2}\right)} = \frac{Sin\left(\frac{m\beta}{2}\right)}{m \times Sin\left(\frac{\beta}{2}\right)}$$

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- Distribution factor for the r^{th} harmonic

$$K_{dr} = \frac{Sin\left(\frac{rm\beta}{2}\right)}{m \times Sin\left(\frac{r\beta}{2}\right)}$$

Q1. An alternator has 9 slots per pole. The coil span is 8 slot. Find pitch factor for fundamental frequency.

Solution: Number of slots per pole, $n = 9$

Angular displacement between the slots,

$$\beta = \frac{180^\circ}{n} = \frac{180^\circ}{9} = 20^\circ \text{ (electrical)}$$

$$\text{Coil span} = \frac{180^\circ \times \text{coil span in terms of slots}}{\text{Number of slots per pole}} = \frac{180 \times 8}{9} = 160^\circ$$

Chording angle, $\alpha = 180^\circ - \text{coil span} = 180^\circ - 160^\circ = 20^\circ$

Pitch factor for the fundamental frequency

$$k_p = \text{Cos}\frac{\alpha}{2} = \text{Cos}\frac{20^\circ}{2} = 0.9848 \textbf{ Ans.}$$

Figure 28 : M1-T1-Q1(Ans.)

Q2. Calculate the distribution factor for a 3-phase distributed single layer winding of the armature of an alternator with 2 poles and 18 slots.

Solution: Number of slots per pole, $n = \frac{18}{2} = 9$

Number of slots per pole per phase,

$$m = \frac{n}{\text{Number of phases}} = \frac{9}{3} = 3$$

Angular displacement between the slots,

$$\beta = \frac{180^\circ}{n} = \frac{180^\circ}{9} = 20^\circ \text{ (electrical)}$$

Distribution factor,

$$k_d = \frac{\text{Sin } \frac{m\beta}{2}}{m \sin \frac{\beta}{2}} = \frac{\text{Sin } \frac{3 \times 20^\circ}{2}}{3 \sin \frac{20^\circ}{2}} = \frac{1}{3} \frac{\sin 30^\circ}{\sin 10^\circ} = 0.96 \textbf{ Ans.}$$

Figure 29 : M1-T1-Q2(Ans.)

Q3. An 8-pole, 3-phase, 60^o spread, double layer winding has 72 coils in 72 slots. The coils are short pitched by two slots. Calculate winding factor for the fundamental and third harmonic.

Solution: Number of slots per pole, $n = \frac{72}{8} = 9$

Number of slots per pole per phase,

$$m = n \times \frac{\text{phase spread}}{180} = 9 \times \frac{60}{180} = 3$$

Angular displacement between the slots,

$$\beta = \frac{180^\circ}{n} = \frac{180^\circ}{9} = 20^\circ \quad \text{(electrical)}$$

$$\text{Coil span} = \frac{180^\circ \times \text{coil span in terms of slots}}{\text{Number of slots per pole}}$$

$$= \frac{180\,(9-2)}{9} = 140^\circ$$

Chording angle, $\alpha = 180^\circ - \text{coil span} = 180^\circ - 140^\circ = 40^\circ$

Figure 30 : M1-T1-Q3(Ans.)

M1 - Tutorial 1

For the fundamental component

$$\text{Distribution factor, } K_d = \frac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}} = \frac{\sin\frac{3\times 20^\circ}{2}}{3\sin\frac{20^\circ}{2}} = 0.96$$

$$\text{Pitch factor, } K_p = \cos\frac{\alpha}{2} = \cos\frac{40^\circ}{2} = 0.94$$

$$\text{Winding factor, } K_w = K_d \times K_p = 0.96 \times 0.94 = 0.9 \textbf{ Ans.}$$

For the third harmonic component (i.e. $r = 3$)

$$\text{Distribution factor, } K_{d_3} = \frac{\sin\frac{rm\beta}{2}}{m\sin\frac{r\beta}{2}} = \frac{\sin\frac{3\times 3\times 20}{2}}{3\sin\frac{3\times 20^\circ}{2}} = 0.666$$

$$\text{Pitch factor, } K_{p_3} = \cos\frac{r\alpha}{2} = \cos\frac{3\times 40^\circ}{2} = 0.5$$

$$\text{Winding factor, } K_{w_3} = K_{d_3} \times K_{p_3} = 0.666 \times 0.5 = 0.333 \textbf{ Ans.}$$

Figure 31 : M1-T1-Q3(Ans.)

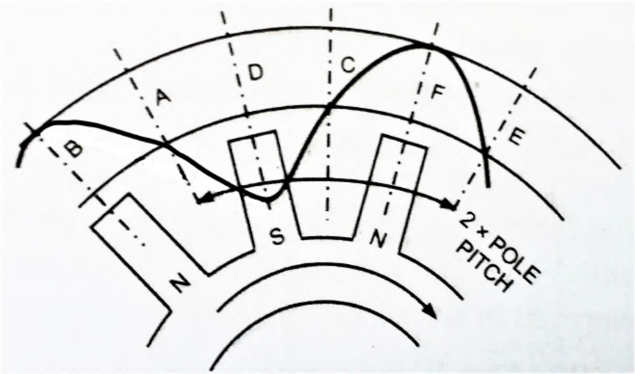

Figure 32 : Production of Alternating emf

Let

- Z_p = No. of conductors in series per phase
- P = No. of Poles
- Φ = Useful flux per pole in webers
- N = Speed of rotation in rpm
- f = Frequency in Hz = $\frac{NP}{120}$

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8. EMF Equation

Let

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- Flux cut by any conductor while passing from centre of one interpolar gap to the centre of next is Φ webers
  $\rightarrow$ emf completes half cycle
  $\implies$ time taken = $\frac{1}{2f}$ seconds
- Average rate of change of flux

$$\frac{d\Phi}{dt} = \frac{\Phi}{\left(\frac{1}{2f}\right)} = 2f\Phi \qquad volts$$

Let

- Average emf per phase

Let

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  E_{av}/phase = No. of conductors in series per phase $\times$ average emf induced per conductor

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- For distributed winding, average emf value of emf per phase will be K_d times the actual value

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  where Turns(T)=$\frac{Z_p}{2}$

- For distributed winding, average emf value of emf per phase will be K_d times the actual value

$$E_{av}/phase = 4K_d\Phi fT \quad volts$$

- For short pitched winding, average value of emf per phase will be

$$E_{av}/phase = 4K_pK_d\Phi fT \quad volts$$

Let

- RMS value of emf per phase

$$E_{rms}/phase = Form\ Factor(K_f) \times E_{av}/phase = 4K_f K_p K_d \Phi f T \qquad volts$$

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- RMS value of emf per phase

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- For sinusoidal wave of emf, $K_f = 1.11$

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- For sinusoidal wave of emf, $K_f = 1.11$

$$\therefore E_{rms}/phase = 4.44 K_p K_d \Phi f T \quad volts$$

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- If alternator is star connected,

$$Line\ induced\ emf,\ E_L = \sqrt{3} \times 4.44 K_p K_d \Phi f T \quad volts$$

Let

- RMS value of emf per phase

$$E_{rms}/phase = Form\ Factor(K_f) \times E_{av}/phase = 4K_f K_p K_d \Phi f T \quad volts$$

- For sinusoidal wave of emf, $K_f = 1.11$

$$\therefore E_{rms}/phase = 4.44 K_p K_d \Phi f T \quad volts$$

- If alternator is star connected,

$$Line\ induced\ emf,\ E_L = \sqrt{3} \times 4.44 K_p K_d \Phi f T \quad volts$$

- For full pitched and concentrated winding, $K_p = 1$ and $K_d = 1$

Q1. A 4-pole alternator has an armature with 25 slots and 8 conductors per slot and rotates at 1500rpm. Flux per pole is 0.05Wb. Calculate the emf generated if winding factor is 0.96 and all conductors are connected in series.

M1 - Tutorial 2

Solution: Flux per pole, $\Phi = 0.05$ Wb

$$\text{Frequency}, f = \frac{PN}{120} = \frac{4 \times 1{,}500}{120} = 50 \text{ Hz}$$

Number of conductors connected in series,

$$Z_p = \text{Number of slots} \times \text{number of conductors per slot}$$

$$= 25 \times 8 = 200$$

$$\text{Number of turns, } T = \frac{Z_p}{2} = \frac{200}{2} = 100$$

$$\text{Winding factor, } K_w = K_d\, K_p = 0.96$$

$$\text{Generated emf, } E = 4.44 \times K_w \times \Phi \times f \times T \text{ volts}$$

$$= 4.44 \times 0.96 \times 0.05 \times 50 \times 100 = 1{,}065.6 \text{ V}$$ **Ans.**

Figure 33 : M1-T2-Q1(Ans.)

Q2. A 3-phase, 50Hz, 20-pole salient pole alternator with star connected stator winding has 180 slots on the stator. Each slot consists of 8 conductors. The flux per pole is 25mWb and is sinusoidally distributed. The coils are full pitched. Calculate

1. The speed of the alternator
2. Winding factor
3. Generated emf per phase
4. Line voltage

Solution:

$$\text{Flux per pole, } \Phi = 25 \text{ mWb} = 0.025 \text{ Wb}$$

$$\text{Frequency, } f = 50 \text{ Hz}$$

$$\text{Number of armature conductors, Z} = 180 \times 8 = 1{,}440$$

$$\text{Number of armature conductors per phase} = \frac{1{,}440}{3} = 480$$

$$\text{Number of turns per phase, T} = \frac{480}{2} = 240$$

$$\text{Number of poles, P} = 20$$

(*i*) $$\text{Speed, N} = \frac{120\,f}{\text{P}} = \frac{120 \times 50}{20} = 300 \text{ rpm}$$ **Ans.**

Figure 34 : M1-T2-Q2(Ans.)

(ii) Number of slots per pole, $n = \frac{180}{20} = 9$

Number of slots per pole per phase,

$$m = \frac{n}{\text{Number of phases}} = \frac{9}{3} = 3$$

Angular displacement between the slots,

$$\beta = \frac{180^\circ}{n} = \frac{180^\circ}{9} = 20^\circ \text{ (elec.)}$$

$$\text{Distribution factor, } K_d = \frac{\text{Sin}\frac{m\beta}{2}}{m\sin\frac{\beta}{2}} = \frac{\text{Sin}\frac{3\times 20^\circ}{2}}{3\sin\frac{20^\circ}{2}} = \frac{\text{Sin } 30^\circ}{3 \sin 10^\circ} = 0.96$$

Pitch factor, $K_p = 1$ $\quad \because$ Coils are full-pitched

(ii) Winding factor, $K_W = K_d K_p = 0.96 \times 1 = 0.96$ **Ans.**

(iii) Generated emf per phase $= 4.44\, K_d K_p\, \Phi f T$ volts

$= 4.44 \times 0.96 \times 1 \times 0.025 \times 50 \times 240$

$= 1.280$ V **Ans.**

(iv) Line voltage, $V_L = \sqrt{3} \times 1{,}280 = 2{,}215$ V **Ans.**

Figure 35 : M1-T2-Q2(Ans.)

Q3. Calculate the rms value of the induced emf per phase of a 10-pole, three phase, 50Hz alternator with 2 slots per pole per phase and 4 conductors per slot in two layers. The coil span is 150^o. The flux per pole is 0.12Wb

Solution: Number of slots per pole per phase, $m = 2$

Number of slots per pole, $n = m \times$ number of phases $= 2 \times 3 = 6$

Number of slots per phase $= m \times$ number of poles $= 2 \times 10 = 20$

Number of conductors connected in series per phase,

$Z_p =$ Number of conductors per slot $\times$ number of slots per phase

$= 4 \times 20 = 80$

$$\text{Number of turns per phase, T} = \frac{Z_p}{2} = \frac{80}{2} = 40$$

$$\text{Flux per pole, } \Phi = 0.12 \text{ Wb}$$

$$\text{Supply frequency, } f = 50 \text{ Hz}$$

Angular displacement between slots,

$$\beta = \frac{180^\circ}{n} = \frac{180^\circ}{6} = 30^\circ$$

$$\text{Distribution factor, } K_d = \frac{\text{Sin}\frac{m\beta}{2}}{m \sin\frac{\beta}{2}} = \frac{\text{Sin}\frac{2\times 30^\circ}{2}}{2 \sin\frac{30^\circ}{2}} = 0.966$$

$$\text{Chording angle, } \alpha = 180^\circ - \text{coil span} = 180^\circ - 150^\circ = 30^\circ$$

$$\text{Pitch factor, } K_p = \text{Cos}\frac{\alpha}{2} = \text{Cos}\frac{30^\circ}{2} = 0.966$$

RMS value of induced emf per phase,

$$E_{p\,(rms)} = 4.44 K_d K_f \times \Phi \times f \times T$$

$$= 4.44 \times 0.966 \times 0.966 \times 0.12 \times 50 \times 40$$

$$= 994.37 \text{ V} \quad \textbf{Ans.}$$

Figure 36 : M1-T2-Q3(Ans.)

Q4. Calculate the rms value of the induced emf per phase of a 10-pole, 3-phase, 50Hz alternator with 2 slots per pole per phase and 4 conductor per slot in two layers. The coil span is 150^o. The flux per pole has a fundamental component of 0.12Wb and a 20% third harmonic component.

Solution: Number of slots / pole / phase, $m = 2$

Number of slots / pole, $n = 2 \times 3 = 6$

Number of slots / phase $= 2 \times 10 = 20$

Number of conductors connected in series, $Z_p = 20 \times 4 = 80$

Number of series turns/phase, $T = \frac{Z_p}{2} = \frac{80}{2} = 40$

Angular displacement between adjacent slots,

$$\beta = \frac{180^\circ}{n} = \frac{180^\circ}{6} = 30^\circ$$

$$\text{Distribution factor, } K_d = \frac{\text{Sin } m\frac{\beta}{2}}{m\sin\frac{\beta}{2}} = \frac{\text{Sin } \frac{2\times 30^\circ}{2}}{\text{Sin } \frac{30^\circ}{2}} = 0.966$$

$$\text{Coil span factor, } K_p = \text{Cos } \frac{\alpha}{2} = \text{Cos } \frac{180^\circ - 150^\circ}{2}$$

$$= \text{Cos } 15^\circ = 0.966$$

Induced emf per phase (fundamental component),

$$E_1 = 4.44\, K_d\, K_p\, \Phi f T$$

$$= 4.44 \times 0.966 \times 0.966 \times 0.12 \times 50 \times 40$$

$$= 994.4 \text{ V.}$$

Figure 37 : M1-T2-Q4(Ans.)

For third harmonic component of flux

$$\text{Distribution factor, } K_{d_3} = \frac{\text{Sin}\,\frac{mr\beta}{2}}{m\sin\frac{r\beta}{2}} = \frac{\text{Sin}\,\frac{2\times3\times30^\circ}{2}}{2\sin\frac{3\times30^\circ}{2}} = 0.707$$

$$\text{Coil span factor, } K_{p_3} = \text{Cos}\,3\times\frac{180^\circ-150^\circ}{2} = \text{Cos}\,45^\circ = 0.707$$

$$\text{Frequency, } \Phi_3 = 3\times f = 3\times50 = 150$$

$$\text{Flux per pole, } \Phi_3 = \frac{1}{3}\times0.12\times\frac{20}{100} = 0.008\text{ Wb}$$

Induced emf per phase (third harmonic component)

$$E_3 = 4.44\,K_d\,K_p\,\Phi_3 f_3\,T$$
$$= 4.44\times0.707\times0.707\times0.008\times150\times40 = 106.56\text{ V}$$

Induced emf per phase,

$$E_p = \sqrt{E_1^2+E_3^2} = \sqrt{(994.4)^2+(106.56)^2} = 1{,}000\text{ V } \textbf{Ans.}$$

Figure 38 : M1-T2-Q4(Ans.)

Wave Shape

- Flux distribution curve in the air gap $\rightarrow$ **not perfectly sinusoidal**

- Flux distribution curve in the air gap $\rightarrow$ **not perfectly sinusoidal** (unless the machine has salient pole rotor with special shaped pole shoes graduating the air gap or a cylindrical rotor with a sinusoidally distributed field winding)
- **Induced emf wave per conductor is similar to flux wave**

Wave Shape

- Flux distribution curve in the air gap $\rightarrow$ **not perfectly sinusoidal** (unless the machine has salient pole rotor with special shaped pole shoes graduating the air gap or a cylindrical rotor with a sinusoidally distributed field winding)
- **Induced emf wave per conductor is similar to flux wave** since induced emf per conductor $\propto$ flux density(B) at constant speed

Wave Shape

- Flux distribution curve in the air gap $\rightarrow$ **not perfectly sinusoidal** (unless the machine has salient pole rotor with special shaped pole shoes graduating the air gap or a cylindrical rotor with a sinusoidally distributed field winding)
- **Induced emf wave per conductor is similar to flux wave** since induced emf per conductor $\propto$ flux density(B) at constant speed ($\because$ $e{=}Blv$ volts, where l(length of conductor) is constant and at constant speed, v is also constant)

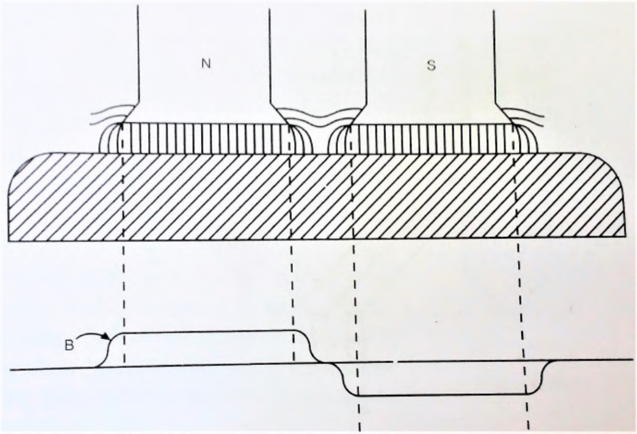

Figure 39 : Waveform of flux density in airgap

Wave Shape

- If winding is full pitched, the emfs induced in two coil sides of each coil will be in phase and of the same magnitude because

Wave Shape

- If winding is full pitched, the emfs induced in two coil sides of each coil will be in phase and of the same magnitude because at any instant both coil sides lie under corresponding positions of opposite poles

Wave Shape

- If winding is full pitched, the emfs induced in two coil sides of each coil will be in phase and of the same magnitude because at any instant both coil sides lie under corresponding positions of opposite poles
- $\implies$ Induced emf wave in each coil will have same shape as the emf induced in each coil side.
- If winding is concentrated $\rightarrow$ resultant emf wave will have same shape as flux density(B) curve.

Wave Shape

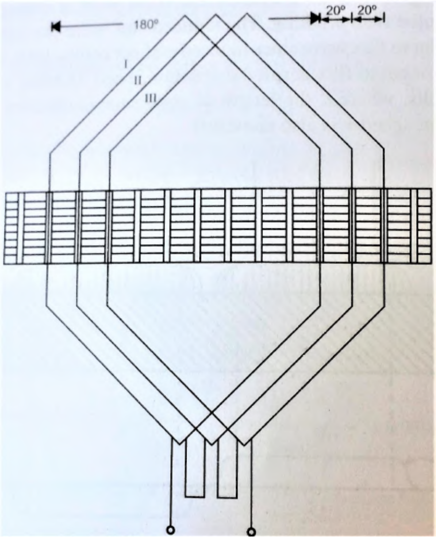

Figure 40 : Winding

Wave Shape

Consider the winding $\rightarrow$ 3-coils, 3-phase, 3-slots per pole per phase

- Full pitched winding
- Displacement angle between adjacent slots $\beta = 20^o$ electrical
- Induced emf waveform of coils are represented by curves I, II and III

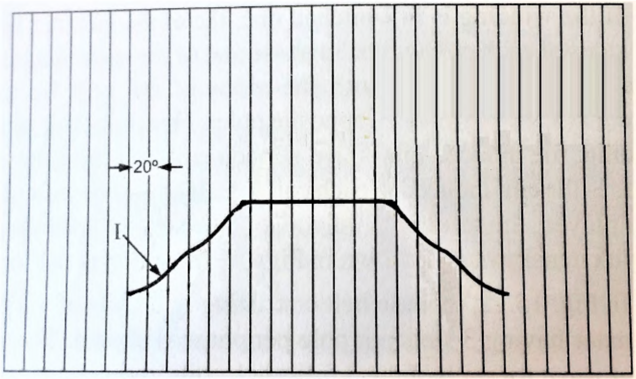

Figure 41 : emf waveform - Coil I

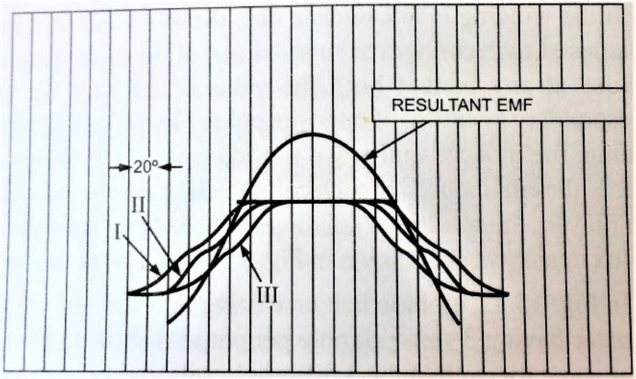

Figure 42 : emf waveform

- Individual emfs are not sinusoidal(Flat topped)

Wave Shape

- Individual emfs are not sinusoidal(Flat topped)
- Resultant emf per phase is almost sinusoidal

Wave Shape

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- Distribution factor(K_d) for harmonics is much less than for the fundamental.
- Hence harmonics are reduced

Wave Shape

For short pitch winding

- emf of coil = phasor sum of emfs of each coil side
- Resultant emf per phase = phasor sum of each coil emf
- Pitch factor(K_p) for harmonics is much less than for the fundamental component
- Hence harmonics are reduced
- Resultant emf will be more sinusoidal

Sources of harmonics in voltage waveform

Sources of harmonics in voltage waveform

1. Non-sinusoidal waveform of field flux

Sources of harmonics in voltage waveform

1. Non-sinusoidal waveform of field flux
2. Variation of reluctance of air gap due to slotting of stator core

Suppression of Harmonics

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- **Salient Pole Alternator** - If air gap is made to vary sinusoidally around the machine $\rightarrow$ flux distribution would be sinusoidal

Suppression of Harmonics

- **Salient Pole Alternator** - If air gap is made to vary sinusoidally around the machine $\rightarrow$ flux distribution would be sinusoidal
  $\Longrightarrow$ increase air gap gradually from centre towards the pole tip

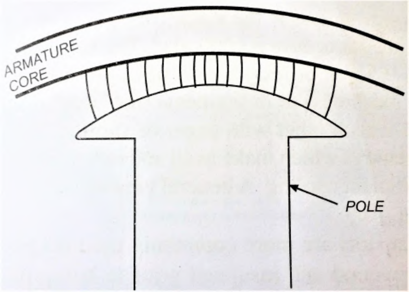

Figure 43 : Pole

- Sinusoidal field $\rightarrow$ Sinusoidal voltage waveform
- Sinusoidal field can also be obtained by skewing of pole faces

- **Cylindrical or Non-salient pole Alternator** - Length of air gap is uniform throughout the periphery
  - Hence mmf of field winding is made to vary as nearly sinusoidal as possible by distributing the windings in slots

9. Harmonics in Voltage Waveform

Harmonics in the induced emf can be eliminated by properly designing the winding

- **Distributed Armature Winding**
- **Short Pitched Armature Winding**
- **Skewing of poles** - usually through one slot pitch

Figure 44 : Skewing of slots

- **Fractional Slot Winding** - Higher order harmonics can be effectively eliminated
- **Larger Air Gap Length** - causes increase in reluctance which will reduce harmonics

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Thank You

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Synchronous and Induction Machines

Synchronous and Induction Machines (EE-202)

by

Prof. Dinto Mathew
Asst. Professor
Dept. of EEE, MACE

Module 2 - Overview

1. Armature Reaction
   - Armature Reaction at Unity Power Factor, Lagging Zero Power Factor and Leading Zero Power Factor
   - Armature Leakage Reactance, Armature Resistance, Synchronous Reactance and Synchronous Impedance
2. Alternator on Load
   - Alternator on Load - Phasor Diagram
   - Load Characteristic of Alternator
3. Experimental Determination
   - Effective Resistance of Armature
   - Open Circuit Test
   - Short Circuit Test
4. Voltage Regulation
   - Synchronous Impedance Method or EMF Method
   - Ampere-Turn Method or MMF Method
   - Zero Power Factor or Potier Method
   - ASA(American Standards Association) Method

Leakage Reactance

- Current in stator conductor $\rightarrow$ flux is set up

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- Current in stator conductor $\rightarrow$ flux is set up
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- Leakage Flux(Φ_L) depends on Stator(Armature) Current and phase angle between stator voltage and current

Leakage Reactance

- Current in stator conductor $\rightarrow$ flux is set up
- Portion of flux that does not cross air-gap $\rightarrow$ completes its path in stator $\rightarrow$ **Leakage Flux**
- Leakage Flux(Φ_L) depends on Stator(Armature) Current and phase angle between stator voltage and current
- Φ_L sets up emf of self-inductance leading load current(I) by $\pi/2$ and proportional to load current(I) in magnitude

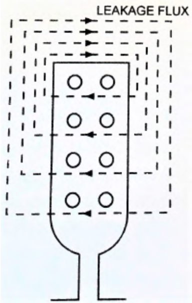


Figure 1 : Leakage Flux

- $\therefore$ Armature winding is assumed to posses leakage reactance(X_L) such that voltage drop due to it ie, (IX_L) is equal to an emf set up by leakage flux
- Part of generated emf is used to overcome the leakage reactance drop

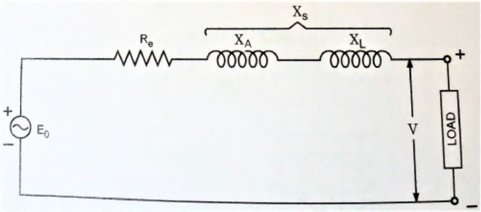


Figure 2 : Equivalent Circuit Diagram

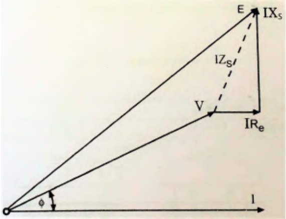
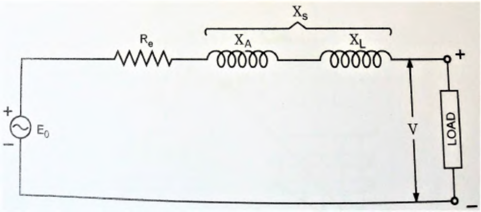


Figure 3 : Phasor Diagram



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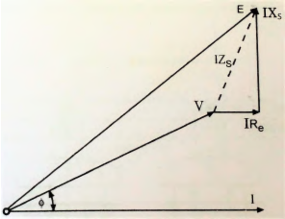

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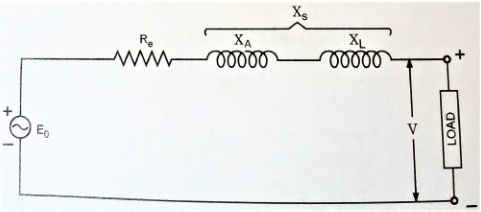
- Generated emf = Phasor sum of terminal voltage, armature resistance drop and leakage reactance drop



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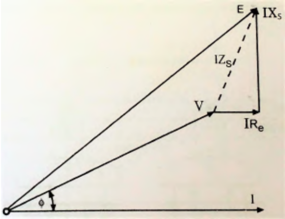

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- Generated emf = Phasor sum of terminal voltage, armature resistance drop and leakage reactance drop

$$\implies \vec{E_o} = \vec{V} + \vec{I} R_e + j \vec{I} X_s = \vec{V} + \vec{I} \left(R_e + j \left(X_L + X_a \right) \right)$$

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- **Armature Reaction** - The effect of magnetic field set up by the armature current on main field flux

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- Phase displacement(between E and I) : $-\pi/2 \leq \Phi \leq +\pi/2$

Three cases :

1. **Case a:**

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Three cases :

1. **Case a:** When $\Phi = 0; \implies$ Unity Power Factor

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Three cases :

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2. **Case b:**

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1. **Case a:** When $\Phi = 0; \implies$ Unity Power Factor
2. **Case b:** When $\Phi = +\pi/2; \implies$ Zero Power Factor Lagging

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3. **Case c:**

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2. **Case b:** When $\Phi = +\pi/2; \implies$ Zero Power Factor Lagging
3. **Case c:** When $\Phi = -\pi/2; \implies$ Zero Power Factor Leading

1.1 Case (a) : At Unity Power Factor

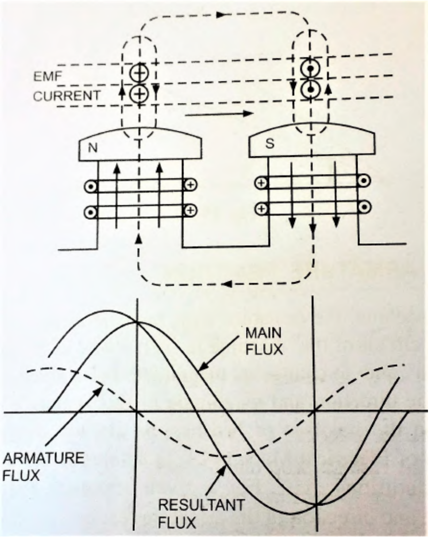

Figure 4 : Armature Reaction at Unity Power Factor

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- $\otimes$ and $\odot$ indicate the instantaneous direction of induced emf and current in armature($\otimes \rightarrow$ Inward direction and $\odot \rightarrow$ Outward direction)
- Assume clockwise rotation of rotor

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- $\implies$ **Asymmetrical distribution of flux density under pole shoe**

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- **Flux density under trailing pole tip increases while under leading pole tip decreases**
- Axis of resultant field is displaced due to armature reaction in a direction **opposite to that of rotation of rotor**
- **Armature Reaction at Unity Power Factor has Distorting Effect**

1.1 Case (b) : At Lagging Zero Power Factor

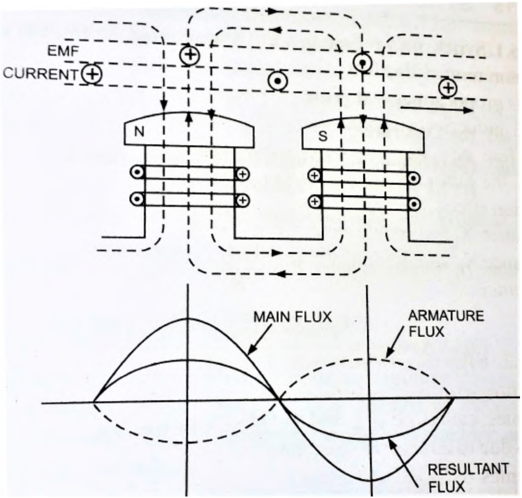

Figure 5 : Armature Reaction at Lagging Zero Power Factor

- Phase angle $\Phi = +\pi/2$(Purely Inductive Load)

1.1 Case (b) : At Lagging Zero Power Factor

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- Field flux due to armature reaction mmf is in the **opposite direction** to main field flux
- **Armature Reaction at Lagging Zero Power Factor has wholly Demagnetising Effect**

1.1 Case (c) : At Leading Zero Power Factor

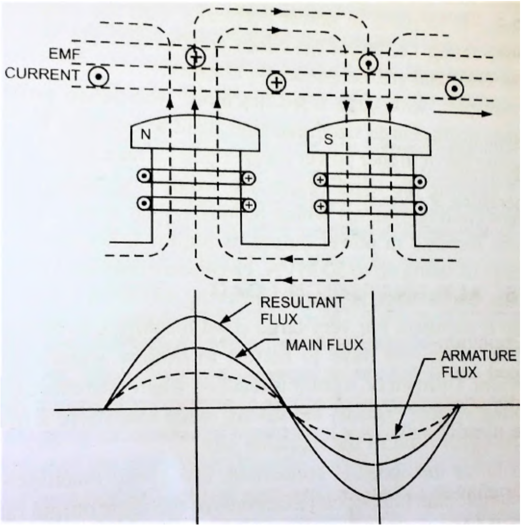

Figure 6 : Armature Reaction at Leading Zero Power Factor

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- Field flux due to armature reaction mmf is in the **same direction** to main field flux
- **Armature Reaction at Leading Zero Power Factor has wholly Magnetising Effect**

- In general, for any power factor(CosΦ) of load, armature reaction has

  * **cross-magnetising component** $\propto$ **I cosΦ** and

  * **demagnetising component** $\propto$ **ISinΦ**(+ve for lagging pf and -ve for leading pf)

- DC resistance
- Additional losses owing to alternating current $\rightarrow$ eddy currents and hysteresis in surrounding material, unequal current distribution in conductor etc.
- Measure armature resistance using direct current and increase it to a higher value $\rightarrow$ **Effective Armature Resistance**(R_e)

1.2 Effective Armature Resistance(R_e)

- DC resistance
- Additional losses owing to alternating current $\rightarrow$ eddy currents and hysteresis in surrounding material, unequal current distribution in conductor etc.
- Measure armature resistance using direct current and increase it to a higher value $\rightarrow$ **Effective Armature Resistance**(R_e)
- $R_e = (1.25 \;\; to \;\; 1.75)$ times R_{dc}

1.2 Synchronous Reactance(X_s)

- emf set up due to armature reaction mmf(F_a) is in quadrature with load current(I) and proportional to it
- Effect of armature reaction is equivalent to reactance drop IX_a
- X_a = Fictitious reactance which takes care of armature reaction effect
- **Synchronous Reactance = Leakage Reactance + Armature Reactance**
- $X_s = X_L + X_a$

- Synchronous reactance combined with effective armature resistance $\rightarrow$ synchronous impedance
- $Z_s = R_e + jX_s$
- $R_e << X_s$. Hence$Z_s \approxeq X_s$

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- The change in terminal voltage of an alternator with change in load is due to

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2. Voltage drop on account of armature leakage reactance

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- The change in terminal voltage of an alternator with change in load is due to

1. Voltage drop on account of effective armature resistance
2. Voltage drop on account of armature leakage reactance
3. Voltage drop on account of armature reaction

Equivalent Circuit of Alternator

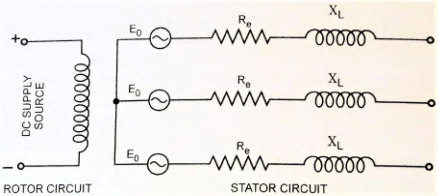


Figure 7 : Circuit Diagram

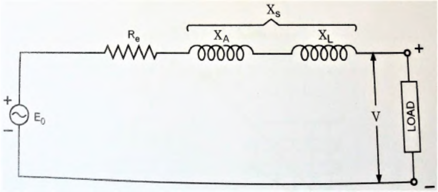
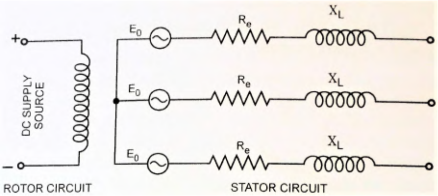


Figure 8 : Equivalent Circuit Diagram

- E_o = Excitation or Generated emf per phase
- At no-load,
  Terminal voltage per phase(V) = Generated emf per phase(E_o)



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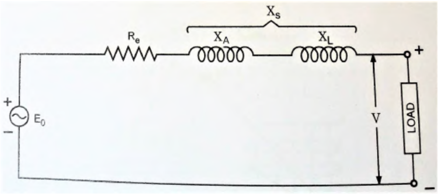

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- E_o = Excitation or Generated emf per phase
- At no-load,
  Terminal voltage per phase(V) = Generated emf per phase(E_o)
- $\overrightarrow{E_o} = \overrightarrow{V} + \overrightarrow{I} R_e + j\overrightarrow{I} X_s$

$$= \overrightarrow{V} + \overrightarrow{I}\,(R_e + j\,(X_L + X_a))$$

$$= \overrightarrow{V} + \overrightarrow{I} Z_s$$

- Let

- R_a = Effective armature resistance
- X_L = Leakage reactance
- X_a = Armature reactance
- V = Terminal voltage per phase(Volts)
- I = Load current per phase(Amperes)
- $Cos\Phi$ = Power factor
- IR_e = Voltage drop due to effective armature resistance
- IX_L = Voltage drop due to leakage reactance
- IX_a = voltage drop due to armature reaction

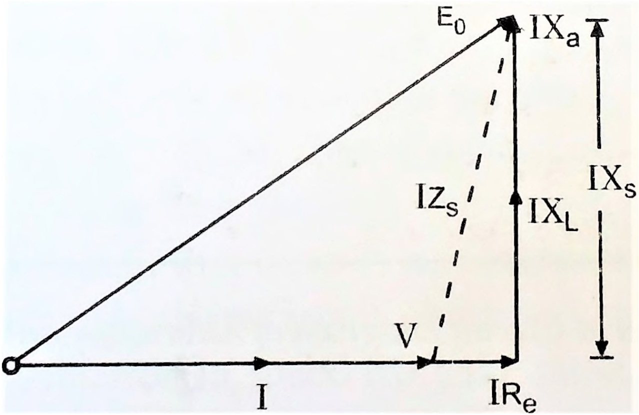

Figure 9 : At Unity pf

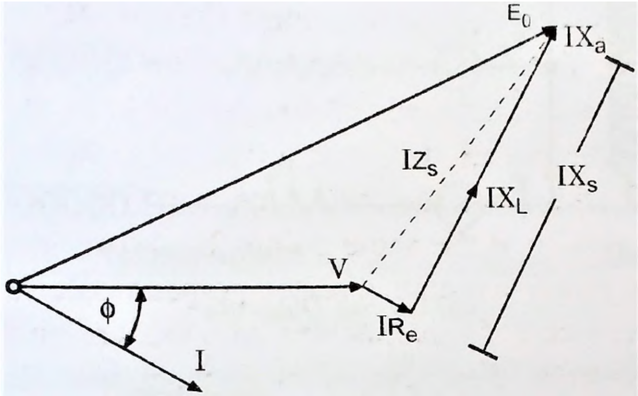

Figure 10 : At Lagging pf

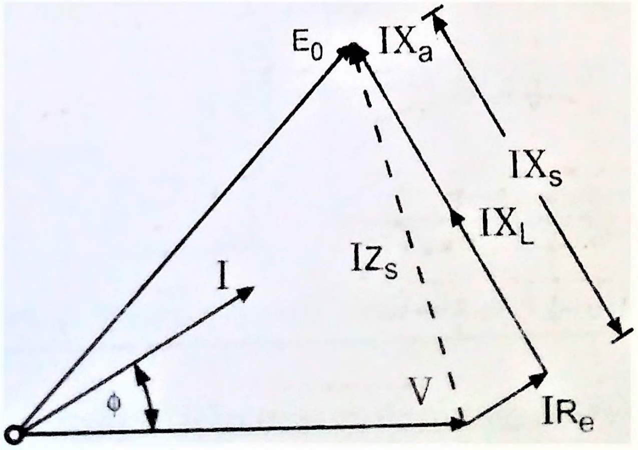

Figure 11 : At Leading pf

2.2 Load Characteristic of Alternator

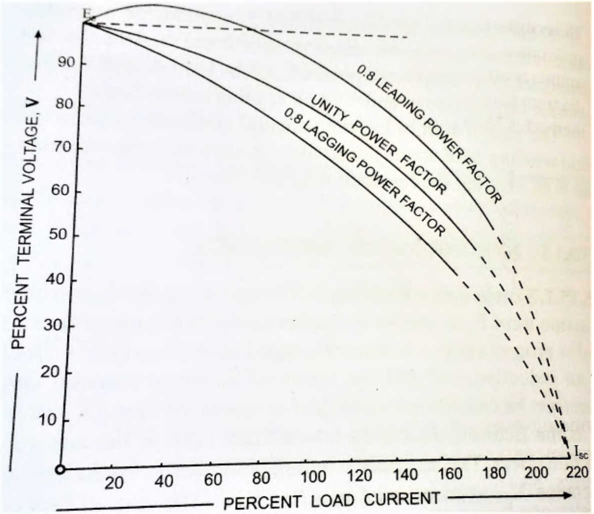

Figure 12 : Load Characteristic of Alternator

- **Load Characteristic** $\rightarrow$ Variation of terminal voltage(V) with load current(I) for constant excitation and speed

2.2 Load Characteristic of Alternator

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- Highest current is obtained when terminals are short circuited

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$$\implies I_{sc} = \frac{E_o}{Z_s}$$

where Z_s = Synchronous Impedance

- Effective Armature Resistance(R_e)
- Synchronous Impedance(Z_s)
- Synchronous Reactance(X_s) $= \sqrt{({Z_s}^2 - {R_e}^2)}$

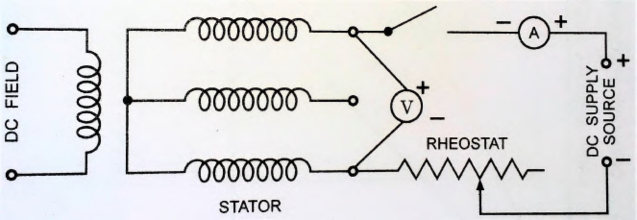


Figure 13 : Circuit Diagram

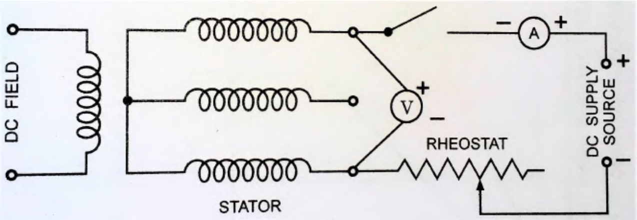
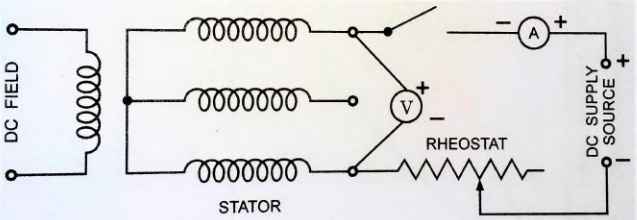


Figure 13 : Circuit Diagram

- R_e can be measured by Voltmeter-Ammeter Method
- DC resistance between a pair of terminals, with field winding open is measured



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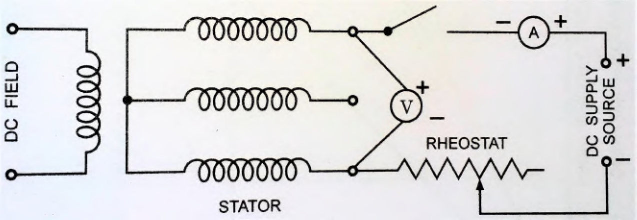

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- DC resistance between a pair of terminals, with field winding open is measured
- Resistance of each phase, $R_a = \left(\frac{V/I}{2}\right)$ ohms
- Effective Armature Resistance per phase$(R_e) = (1.25\ to\ 1.75)\ times\ R_a$
- Usually $R_e = 1.4 \times R_a$

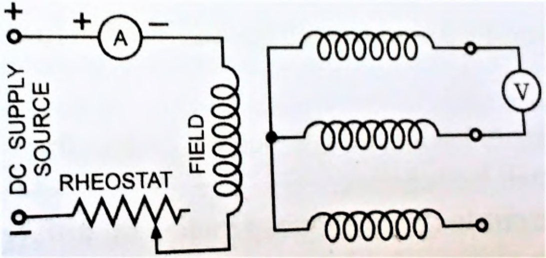

Figure 14 : OC Test on 3-Phase Alternator

3.2 Open Circuit Test

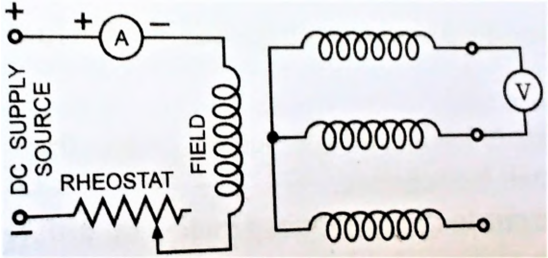

Figure 14 : OC Test on 3-Phase Alternator

- Armature winding is **open circuited**
- Alternator is run at **rated speed**
- Field → Connected to DC source in series with field rheostat and ammeter
- Field current is increased in steps in steps till alternator generates emf above rated emf

- Open circuit voltage per phase$(E_o) = \dfrac{Voltmeter \;\; reading}{\sqrt{3}}$

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- Curve E_o Vs I_f $\rightarrow$ **Open Circuit Characteristic**

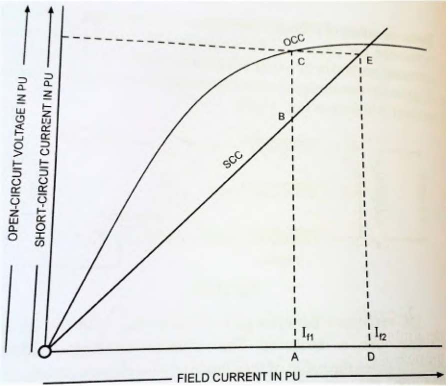

Figure 15 : OCC and SCC

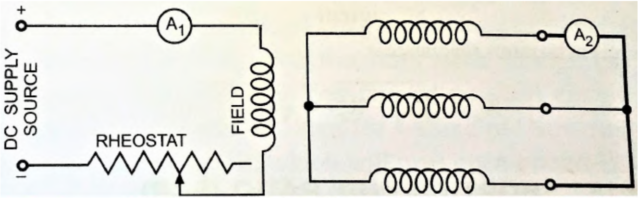

Figure 16 : SC Test on 3-Phase Alternator

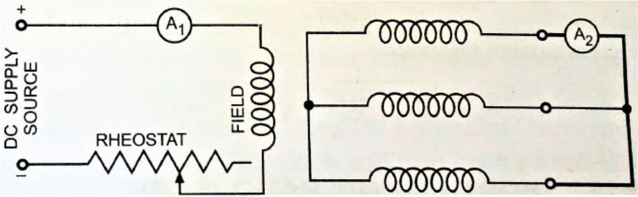

Figure 16 : SC Test on 3-Phase Alternator

- All the three phases are short circuited
- Machine is run at synchronous speed
- Field excitation is raised from zero to an amount sufficient to circulate full load current
- Curve I_{sc} Vs $I_f \rightarrow$ **Short-circuit Characteristic**

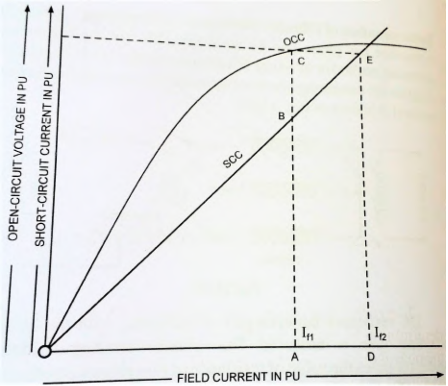

Figure 17 : OCC and SCC

Let

- **OA** $\rightarrow$ field current that gives E_o(represented by 'AC')
- **AB** $\rightarrow$ Short circuit current(I_{sc}) corresponding to field current 'OA'
- $E_o = I_{sc} Z_s$

$$\implies \text{Synchronous Impedance per phase}(Z_s) = \frac{E_o}{I_{sc}} = \frac{AC(in\ volts)}{AB(in\ amperes)}$$

$$\implies \text{Synchronous Reactance}(X_s) = \sqrt{(Z_s^2 - R_e^2)}$$

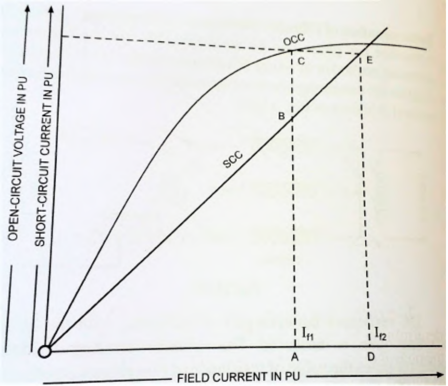

Figure 18 : OCC and SCC

- **Short Circuit Ratio(SCR)** $\rightarrow$ ratio of field current to produce rated voltage on open circuit to field current required to circulate rated current on short circuit while the machine is driven at synchronous speed

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$$\frac{AC}{AB} = \frac{pu\ voltage\ on\ open\ circuit}{Corresponding\ pu\ current\ on\ short\ circuit} = X_s$$

$$SCR = \frac{1}{X_s}$$

- $\implies$ **SCR** is the reciprocal of pu synchronous reactance(X_s)
- Small value of SCR $\rightarrow$ smaller value of current under short-circuit $\rightarrow$ Large value of X_s

Short Circuit Ratio(SCR)

- For turbo-alternators; SCR $\rightarrow$ between 0.5 and 0.75
- For salient pole alternator; SCR $\rightarrow$ between 1 and 1.5
- High value of SCR $\rightarrow$ Stability limit is increased, voltage regulation is improved
- Low value of SCR $\rightarrow$ difficulty in parallel operation since synchronizing power is small

- Terminal voltage of alternator(V) changes from no-load to full load

4. Voltage Regulation

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- Change in terminal voltage depends on
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- **Voltage Regulation** $\rightarrow$ Increase in terminal voltage, expressed as percentage(or pu) of the rated terminal voltage, when the load at a given power factor is thrown-off with speed and field current remaining the same

$$Voltage\ Regulation = \frac{E_o - V}{V}(in\ pu)$$
$$= \frac{E_o - V}{V} \times 100(in\ \%)$$

where E_o = No-load terminal voltage
V = Full load terminal voltage

- For **Leading pf:**

- Terminal voltage of alternator(V) changes from no-load to full load
- Change in terminal voltage depends on
  - Load Current
  - power factor of load
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where E_o = No-load terminal voltage
V = Full load terminal voltage

- For **Leading pf:** $E_o < V, \implies$ Voltage Regulation is **negative**

- Determine R_e and X_s from open-circuit and short-circuit tests
- Draw phasor diagram for any load of any pf

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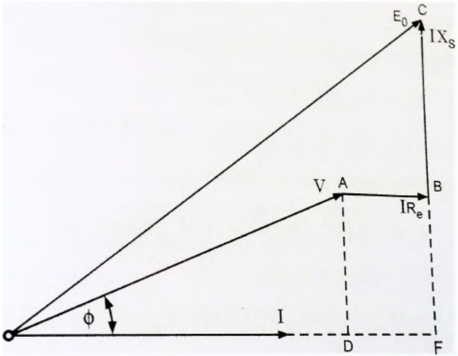


Figure 19 : Phasor Diagram(Lagging pf)

- Let load current(I) lags terminal voltage(V) by an angle(Φ)

$$
\begin{aligned}
No\ load\ terminal\ voltage\ per\ phase(E_o) &= OC \\
Full\ load\ terminal\ voltage(V) &= OA \\
Armature\ resistance\ drop &= I \times R_e = AB \\
Synchronous\ reactance\ drop &= I \times X_s = BC \\
But\ OC^2 &= OF^2 + FC^2 \\
&= (OD + DF)^2 + (FB + BC)^2
\end{aligned}
$$

$$
\begin{aligned}
&\implies E_o^2 = (VCos\Phi + IR_e)^2 + (VSin\Phi + IX_s)^2 \\
&\implies E_o = \sqrt{(VCos\Phi + IR_e)^2 + (VSin\Phi + IX_s)^2} \\
&\implies \%\ Voltage\ Regulation = \frac{E_o - V}{V} \times 100\ \%
\end{aligned}
$$

4.1 Synchronous Impedance Method or EMF Method

- Take Φ as negative for leading pf
- Take $\Phi = 0$ for upf
- Synchronous impedance or EMF method yields inexact regulation value. (However, it is theoretically accurate for non-salient pole machines with distributed windings when saturation is not considered)

- Take Φ as negative for leading pf
- Take $\Phi = 0$ for upf
- Synchronous impedance or EMF method yields inexact regulation value. (However, it is theoretically accurate for non-salient pole machines with distributed windings when saturation is not considered)
- **Calculated Regulation** $>$ **Actual value** $\implies$ **Pessimistic Method** since **synchronous impedance (or reactance) is assumed to remain constant** while actually it is not.
- Synchronous impedance(or reactance) varies with saturation
- At low saturation $\rightarrow$ Z_s (or X_s) is larger(since effect of armature reaction is greater at low saturation than at high saturation)
- Under short-circuit condition, saturation is very low $\rightarrow$ value of synchronous impedance measured is higher than that in actual working conditions

Q1. A three-phase star connected alternator is rated at 1600kVA, 13.5kV. The per phase armature effective resistance and synchronous reactance are 1.5Ω and 30Ω respectively. Calculate voltage regulation for a load of 1.28MW at pf of (i) 0.8pf leading (ii) Unity pf (iii) 0.8pf lagging.

M2 - Tutorial

(i) **At power factor 0.8 leading.**

$$\text{Load current, } I = \frac{\text{Load in MW} \times 10^6}{\sqrt{3}\, V_L \cos\phi} = \frac{1.28 \times 10^6}{\sqrt{3} \times 13.5 \times 10^3 \times 0.8}$$

$$= 68.4 \text{ A}$$

Power factor, $\cos\phi = 0.8$

$$\text{Sin}\,\phi = \sqrt{1 - 0.8^2} = -0.6 \quad \text{minus sign for leading pf}$$

Open-circuit voltage per phase,

$$E_{0P} = \sqrt{(V_P \cos\phi + IR_e)^2 + (V_P \sin\phi + IX_s)^2}$$

$$= \sqrt{(7{,}794 \times 0.8 + 68.4 \times 1.5)^2 + [7{,}794 \times (-0.6) + 68.4 \times 30]^2}$$

$$= 6{,}860 \text{ V}$$

Percentage regulation

$$= \frac{E_{0P} - V_P}{V_P} \times 100 = \frac{6{,}860 - 7{,}794}{7{,}794} \times 100 = -\,11.98\ \% \ \textbf{Ans.}$$

(ii) **At unity power factor**

$$\text{Load current, } I = \frac{1.28 \times 10^6}{\sqrt{3} \times 13.5 \times 10^3 \times 1.0} = 54.74 \text{ A}$$

$$\text{Cos}\,\phi = 1.0 \text{ and } \sin\phi = 0$$

Open-circuit voltage per phase,

$$E_{0P} = \sqrt{(7{,}794 \times 1.0 + 54.74 \times 1.5)^2 + (7{,}794 \times 0 + 54.74 \times 30)^2}$$

$$= \sqrt{(7{,}794 + 82.11)^2 + (0 + 1{,}642.2)^2} = 8{,}045.5 \text{ V}$$

$$\text{Percentage regulation} = \frac{8{,}045.5 - 7{,}794}{7{,}794} \times 100 = 3.227\ \% \ \textbf{Ans.}$$

Figure 20 : M2-T1-Q1(Ans.)

(iii) **At power factor 0.8 lagging**

Load current, $I = 68.4$ A, same as in case *(i)*

$$\text{Cos}\,\phi = 0.8 \text{ and } \sin\phi = 0.6$$

Open-circuit voltage per phase,

$$E_{0P} = \sqrt{(7{,}794 \times 0.8 + 68.4 \times 1.5)^2 + (7{,}794 \times 0.6 + 68.4 \times 30)^2}$$

$$= 9{,}243 \text{ V}$$

$$\text{Percentage regulation} = \frac{9{,}243 - 7{,}794}{7{,}794} \times 100 = 18.6\ \% \ \textbf{Ans.}$$

Figure 21 : M2-T1-Q1(Ans.)

Q2. A full load current of 100A requires an excitation current of 3A when a single-phase 1.2kV alternator is short-circuited by an ammeter of resistance 0.01Ω. On open circuit, the same excitation produces 350V. The resistance of the armature is 0.5Ω. Calculate the % voltage regulation of alternator at 0.8pf leading.

M2 - Tutorial

Solution: Rated voltage, $V = 1{,}200$ V

Rated current, $I = 100$ A

Effective resistance of armature, $R_e = 0.5\ \Omega$

If X_s is the synchronous reactance of the alternator, then

$$\sqrt{(R_e + \text{ammeter resistance})^2 + X_s^2} = \frac{\text{OC voltage}}{\text{SC current}}$$

$$\text{or } \sqrt{(0.5 + 0.01)^2 + X_s^2} = \frac{350}{100} = 3.5\ \Omega$$

$$\text{or } X_s = \sqrt{3.5^2 - 0.51^2} = 3.4626\ \Omega$$

At 0.8 leading pf, $\cos\phi = 0.8$ and $\sin\phi = -0.6$

Open-circuit voltage,

$$E_0 = \sqrt{(V\cos\phi + IR_e)^2 + (V\sin\phi + IX_s)^2}$$

$$= \sqrt{(1{,}200 \times 0.8 + 100 \times 0.5)^2 + (1200 \times (-0.6) + 100 \times 3.4626)^2}$$

$$= 1{,}077 \text{ V}$$

$$\text{Voltage regulation} = \frac{1{,}077 - 1{,}200}{1{,}200} \times 100 = -10.25\% \quad \textbf{Ans.}$$

Figure 22 : M2-T1-Q2(Ans.)

Q3. A 30MVA, 11kV, 50Hz star connected three-phase alternator has an effective armature resistance and synchronous reactance of 0.02Ω and 2.25Ω respectively. Find voltage regulation for a pf of 0.707 lagging. What is the load angle of alternator at this condition.

M2 - Tutorial

Solution: Load, S = 30 MVA

Phase voltage, $V_P = \frac{V_L}{\sqrt{3}} = \frac{11 \times 1{,}000}{\sqrt{3}} = 6{,}351$ V

Effective resistance per phase,

$$R_e = 0.02\ \Omega$$

Synchronous reactance per phase,

$$X_s = 2.25\ \Omega$$

Phase current, I = Line current

$$= \frac{\text{Load MVA} \times 10^6}{\sqrt{3} \times V_L} = \frac{300 \times 10^6}{\sqrt{3} \times 11{,}000}$$

$$= 1{,}574.6 \text{ A}$$

Figure 23 : M2-T1-Q3(Ans.)

$\sin\phi = \sqrt{1 - 0.707^2} = -0.707$ minus sign for lagging pf

Taking V_P as the reference phasor we have

$$\mathbf{V}_P = V_P \angle 0^\circ = 6{,}351 \angle 0^\circ \text{ V}$$

Current, $\mathbf{I} = 1{,}574.6 \angle -45^\circ$ V $\quad \because \phi = \cos^{-1} 0.707 = -45°$

Synchronous impedance per phase,

$$\mathbf{Z}_s = \sqrt{0.02^2 + 2.25^2} = 2.25 \angle 89.49^\circ\ \Omega$$

Open-circuit voltage per phase,

$$\mathbf{E}_{oP} = \mathbf{V} + \mathbf{IZ}_s = 6{,}351 \angle 0^\circ + 1{,}574.6 \angle -45^\circ \times 2.25 \angle 89.49^\circ$$

$$= (6{,}351 + j\,0) + 3{,}542.8 \angle 44.49^\circ$$

$$= (6{,}351 + j\,0) + (2{,}527 + j\,2{,}501)$$

$$= (8{,}878 + j\,2{,}501) = 9{,}223.5 \text{ V} \angle 15.73^\circ \text{ V}$$

So open-circuit voltage per phase,

$$E_{oP} = 9{,}223.5 \text{ V}$$

Percentage voltage regulation $= \frac{E_{oP} - V}{V_P} \times 100$

$$= \frac{9{,}223.5 - 6{,}351}{6{,}351} \times 100 = 45.23\%$$ **An**

Load angle, $\delta = 15.73^\circ$ **Ans.**

Figure 24 : M2-T1-Q3(Ans.)

4.2 Ampere-Turn Method or MMF Method

- Open circuit and short circuit tests are conducted
- From OCC and SCC, I_{f1} and I_{f2} are determined
- $I_{f1} \rightarrow$ field current required to generate rated voltage(V) on no-load
- $I_{f2} \rightarrow$ field current required to circulate short-circuit current, equal to full load current on short circuit
- $I_{f2} \rightarrow$ gives **demagnetising ampere-turns at full load**. (On short-circuit, field excitation(I_{f2}) balances impedance drop($IZ_s \rightarrow IR_e$, IX_L, IX_a). Since R_e is small and X_L is also small for low voltage on short circuit $\rightarrow$ neglect IR_e and $IX_L \implies$ pf on short-circuit is almost **zero pf lagging** and field amp-turns are used entirely to overcome armature reaction)

- Let the alternator supply full load current at a pf of $Cos\Phi$.
- Draw **OA** representing the field current to give full load rated voltage,V

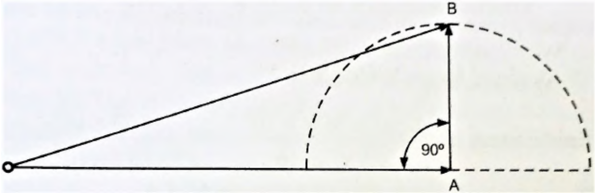


Figure 25 : MMF Method for Unity Power Factor

- Draw **AB** at an angle ($90^o \pm \Phi$ representing the field current to give full load current on short-circuit).
  - +ve sign for lagging pf
  - -ve sign for leading pf
- Measure **OB** representing the field current required to give open-circuit emf E_o which can be determined from OCC

$$I_f = \sqrt{(I_{f1})^2 + (I_{f2})^2 - 2 I_{f1} I_{f2} Cos\,(90 \pm \Phi)}$$

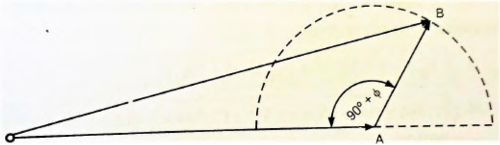


Figure 26 : MMF Method for Lagging Power Factor

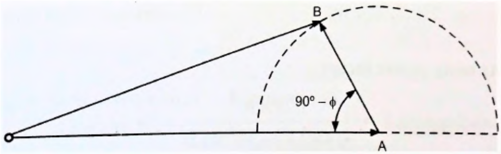


Figure 27 : MMF Method for Leading Power Factor

4.2 Ampere-Turn Method or MMF Method

$$\% \, Voltage \; Regulation = \frac{E_o - V}{V} \times 100 \; \%$$

- Ampere-Turn or MMF Method $\rightarrow$ **Optimistic Method** since it gives voltage regulation value lower than actual value($\because$ excitation to overcome armature reaction is determined on unsaturated part of the saturation curve).

- Based on the separation of reactances due to leakage flux and armature reaction and hence it is more accurate.

The data required are

- Effective armature resistance(R_e)
- Open circuit characteristic(OCC)
- Field current to circulate full load current on short-circuit
- **Zero power factor full-load voltage characteristic** $\rightarrow$ Curve between terminal voltage and field excitation while the machine is being run on synchronous speed and delivering full-load current at zero power factor.

To plot Zero pf full-load curve

- Machine is run at N_s by prime mover
- Purely inductive load(variable load reactor) is connected across the armature terminals
- Field current is increased to circulate full-load armature current
- Then value of reactance is increased step by step $\rightarrow$ field current is adjusted to flow the full-load armature current
- V is varied from 125% to 25%, maintaining speed and rated armature current constant throughout the test.
- Curve between terminal voltage(V) and field current $\rightarrow$ **Zero Power Factor Characteristic Curve**

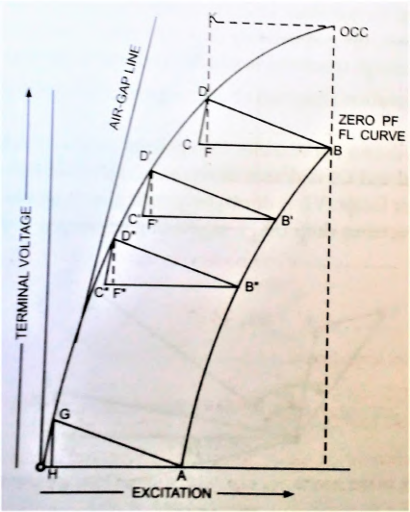


Figure 28 : OCC and ZPF FL curve

4.3 Zero Power Factor or Potier Method

- Zero power factor(lagging) characteristic curve is of exactly the same shape as the OCC but its is shifted vertically **downward by leakage reactance drop(IX_L) and horizontally by armature reaction mmf**
- Hence ZPF(lagging) characteristic curve can be drawn by knowing the points A and B
- Point A $\rightarrow$ from short-circuit test with full load armature current
- OA $\rightarrow$ field current required to overcome demagnetising effect of armature reaction and leakage reactance drop at full load
- Point B $\rightarrow$ When full load current flows through the armature and wattmeter reads zero

To draw **Potier Triangle**

- From point B, BC is drawn equal and parallel to AO
- Line CD is drawn parallel to Air-gap line intersecting OCC at D
- BD is joined
- DF is drawn $\perp$r to BC
- ΔBDF is known as **Potier Triangle**
- In Potier Triangle(ΔBDF),
  - BF$\rightarrow$ Armature reaction excitation
  - DF $\rightarrow$ Leakage reactance drop(IX_L)

$$Potier\ Reactance\,(X_p) = \frac{DF\,(voltage\ drop\ per\ phase)}{Zero\ power\ factor\ current\ per\ phase}$$

- For cylindrical alternator, $X_p \cong X_L$
- For salient pole alternator, $X_p > X_L$

Potier Regulation Diagram

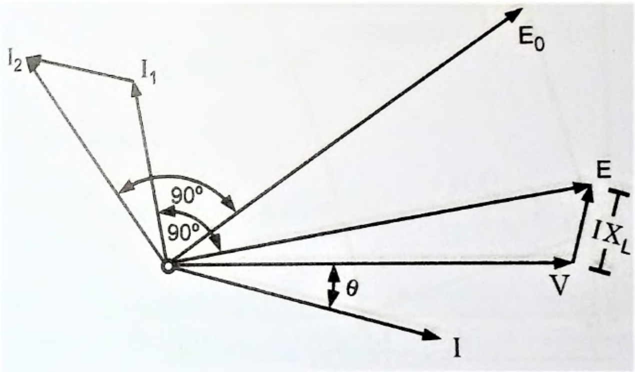


Figure 29 : Potier Regulation Diagram

4.3 Zero Power Factor or Potier Method

- OV $\rightarrow$ Terminal voltage(V) on full-load
- OI $\rightarrow$ full-load current at a given pf
- VE $\rightarrow$ $\perp$r to OI and equal to reactance drop(IX_L), neglecting resistance drop
- OE $\rightarrow$ generated emf(E)
- From OCC, field current(I_1) corresponding to emf(E) is determined
- OI_1 $\perp$r to OE $\implies$ excitation required to induced emf OE on open circuit
- Phasor $I_1 I_2$ is drawn parallel to load current OI $\rightarrow$ excitation equivalent to full-load armature reaction
- OI_2 $\rightarrow$ total excitation
- From OCC, emf E_o corresponding to field excitation OI_2 is determined

$$\% \, Regulation = \frac{E_o - V}{V} \times 100$$

4.4 ASA(American Standards Association) Method

- Modification of MMF Method
- Provides reliable results for salient pole and non-salient pole alternators

Steps

- Determine field current(I_1) corresponding to terminal voltage(V) from air-gap line(It is the excitation current that would be required if the machine had no armature reaction and if there was no saturation of magnetic circuit)
- Determine field current(I_2) required to circulate full-load current on short circuit from SCC
- Draw oa($=I_1$) horizontally
- Draw ab($=I_2$) making an angle ($90\pm\Phi$) from line oa
  - +ve sign for lagging pf generation operation and leading pf motor operation
  - -ve sign for leading pf generation operation and lagging pf motor operation

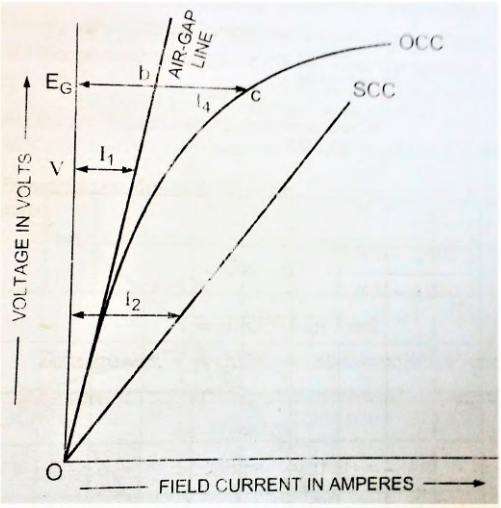


Figure 30 : ASA Method

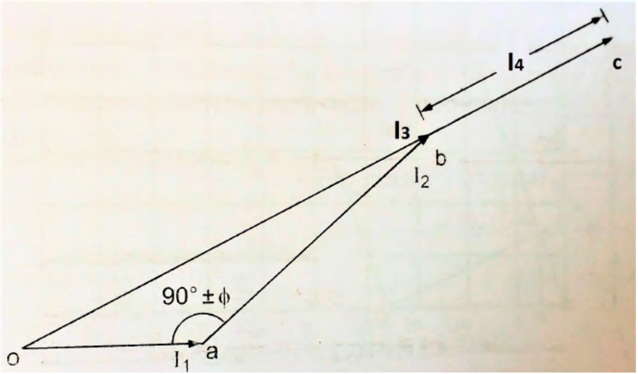


Figure 31 : Phasor

4.4 ASA(American Standards Association) Method

- Field current ob($=I_3$) is the field current that would be required for the considered operating condition provided magnetic circuit remained unsaturated
- E_G is determined by Potier triangle method
- Line parallel to field axis is drawn from E_G to meet OCC at point 'c' and intersecting air-gap line at 'b'
- bc($= I_4$) $\rightarrow$ additional excitation current required because of the saturation of magnetic circuit
- Arithmetic sum of excitation currents $\rightarrow$ excitation current(I_{exc}) required for the operating condition under consideration

$$\implies I_3 + I_4 = I_{exc}$$

$$\%\ Voltage\ Regulation = \frac{E_o - V}{V} \times 100\ \%$$

Q1. An alternator has R_e of 0.3Ω and X_L of 1.22Ω. The alternator supplies 100A current to a feeder of resistance 1.5Ω and reactance of 2Ω. The voltage at the far end of the feeder is 3000V. The load current has a pf of 0.8 lagging with respect to this voltage. Find the terminal voltage of the alternator and the emf generated.

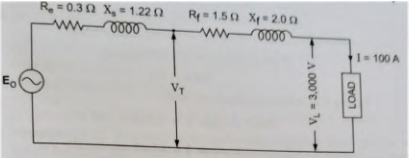


Fig. 15.39

Solution: Load voltage,

$$\mathbf{V}_L = 3{,}000\ \angle 0^\circ \quad \text{Taking } V_L \text{ as reference phasor}$$

Load current. $\mathbf{I}_L = 100\ \angle -36.87^\circ$ A

$$\because\ \phi = \cos^{-1} 0.8 \text{ (lagging)} = -36.87^\circ$$

Feeder impedance,

$$\mathbf{Z}_f = (R_f + j X_f) = (1.5 + j\,2.0)\ \Omega = 2.5\ \angle 53.13^\circ\ \Omega$$

Synchronous impedance of alternator,

$$\mathbf{Z}_s = (R_e + j X_s) = (0.3 + j\,1.22)\ \Omega = 1.256\ \angle 76.18^\circ\ \Omega$$

Figure 32 : M2-T2-Q1(Ans.)

Terminal voltage of alternator

$$\begin{aligned}
V_T &= \text{Load voltage} + \text{Voltage drop in feeder} \\
&= 3{,}000\ \angle 0^\circ + 100\ \angle -36.87^\circ \times 2.5\ \angle 53.13^\circ \\
&= 3{,}000\ \angle 0^\circ + 250\ \angle 16.26^\circ \\
&= (3{,}000 + j0) + (240 + j70) \\
&= (3{,}240 + j70) = 3{,}240.76\ \angle 1.24^\circ \text{ V} \quad \textbf{Ans.}
\end{aligned}$$

Generated emf,

$$\begin{aligned}
\mathbf{E}_0 &= \mathbf{V}_T + \mathbf{I}_L \mathbf{Z}_s \\
&= (3{,}240 + j70) + 100\ \angle -36.87^\circ \times 1.256\ \angle 76.18^\circ \\
&= (3{,}240 + j70) + 125.6\ \angle 39.31^\circ \\
&= (3{,}240 + j70) + (97.18 + j79.57) \\
&= (3{,}337.18 + j149.57) = 3{,}340.5\ \angle 2.57^\circ \text{ V} \quad \textbf{Ans.}
\end{aligned}$$

Figure 33 : M2-T2-Q1(Ans.)

Q2. A three-phase star-connected synchronous generator rated at 10kVA and 230V has X_s of 1.2Ω per phase and R_e of 0.5Ω per phase. Calculate the following. (i) The % voltage regulation at full load and 0.8pf lagging (ii) The pf of load such that the voltage regulation is zero on full load.

M2 - Tutorial

Solution: Phase voltage, $V_p = \frac{230}{\sqrt{3}} = 132.79$ V

Load, S = 10 kVA

Load current, $I = \frac{10 \times 1{,}000}{\sqrt{3} \times 230} = 25.1$ A

Effective resistance per phase, $R_e = 0.5\ \Omega$

Synchronous reactance per phase, $X_s = 1.2\ \Omega$

Power factor, $\cos\phi = 0.8$

$$\sin\phi = \sqrt{1^2 - 0.8^2} = 0.6$$

Open-circuit voltage per phase,

$$E_{oP} = \sqrt{(V_P \cos\phi + IR_e)^2 + (V_P \sin\phi + IX_s)^2}$$

$$= \sqrt{(132.79 \times 0.8 + 25.1 \times 0.5)^2 + (132.79 \times 0.6 + 25.1 \times 1.2)^2}$$

$$= 161.75 \text{ V}$$

Percentage regulation $= \frac{E_{oP} - V_P}{V_P} \times 100 = \frac{161.75 - 132.79}{132.79} \times 100$

$= 21.8\%$ **Ans.**

Figure 34 : M2-T2-Q2(Ans.)

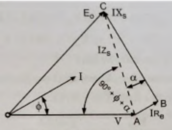


Fig. 15.40 *Phasor Diagram*

In phasor diagram shown in Fig. 15.40.

$$\alpha = \text{Tan}^{-1}\frac{IR_e}{IX_s} = \text{Tan}^{-1}\frac{0.5}{1.2} = 22.62°$$

$$OC = E_0$$

$$OA = V$$

$$AC = \sqrt{(IR_e)^2 + (IX_s)^2}$$

$$= \sqrt{(25.1 \times 0.5)^2 + (25.1 \times 1.2)^2} = 32.63 \text{ V}$$

Now $OC^2 = OA^2 + AC^2 + 2OA \times AC \times \sin(\phi + \alpha)$

or $\sin(\phi + \alpha) = \frac{-AC^2}{2 \times OA \times AC}$

$\because$ For zero regulation $E_0 = V$ or $OC = OA$

$$= \frac{-AC}{2 \times OA} = -\frac{32.63}{2 \times 132.79} = -0.12286$$

or $\phi + \alpha = \text{Sin}^{-1}(-0.12286) = -7.057°$

or $\phi = -7.057° - \alpha = -7.057° - 22.62° = -29.68°$

Power factor $= \text{Cos}\,\phi = \text{Cos}\,(-29.68°) = 0.8688$ leading **Ans.**

Figure 35 : M2-T2-Q2(Ans.)

Q3. A 10kVA, 440V, 50Hz, 3Φ alternator has the following OCC.

Field Current(A)	Terminal Voltage(V)
1	100
3	300
5	440
8	550
11	600
15	635

With full load zero pf load applied, an excitation of 14A produced a terminal voltage of 500V. On short-circuit, 4A excitation was required to circulate full load current. Using mmf method, determine the full load % regulation for 0.6pf lagging and 0.6pf leading

M2 - Tutorial

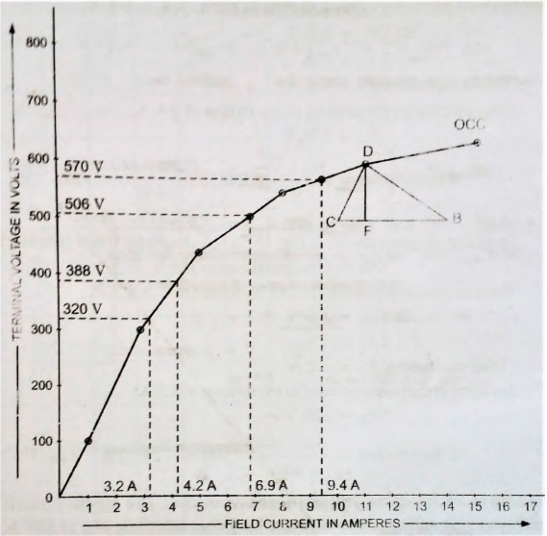
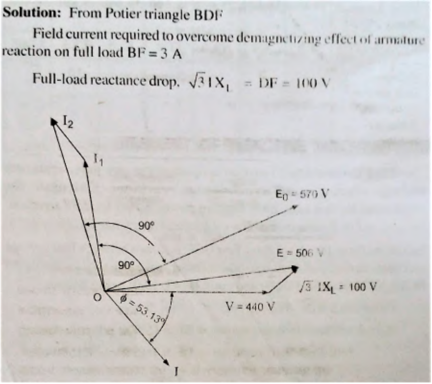


Figure 36 : M2-T2-Q3(Ans.)

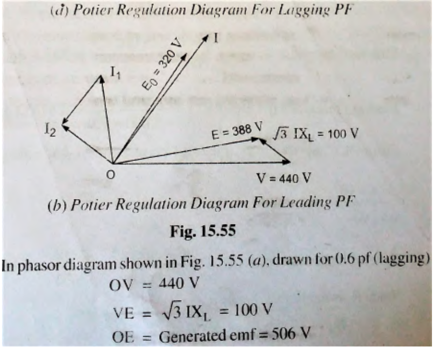


Solution: From Potier triangle BDF

Field current required to overcome demagnetizing effect of armature reaction on full load BF = 3 A

Full-load reactance drop, $\sqrt{3}\,IX_L = DF = 100$ V

Figure 37 : M2-T2-Q3(Ans.)



(*a*) *Potier Regulation Diagram For Lagging PF*

(*b*) *Potier Regulation Diagram For Leading PF*

Fig. 15.55

In phasor diagram shown in Fig. 15.55 (*a*), drawn for 0.6 pf (lagging)

$$OV = 440 \text{ V}$$
$$VE = \sqrt{3}\,IX_L = 100 \text{ V}$$
$$OE = \text{Generated emf} = 506 \text{ V}$$

Figure 38 : M2-T2-Q3(Ans.)

Excitation corresponding to 506 V, from OCC is 6.9 A

$$\text{So } OI_1 = 6.9 \text{ A}$$

$$I_1I_2 = 3 \text{ A}$$

$$\text{Total excitation } OI_2 = 9.4 \text{ A}$$

Induced emf corresponding to excitation of 9.4 A,

$$E_0 = 570 \text{ V}$$

$$\% \text{ Regulation} = \frac{E_0 - V}{V} \times 100 = \frac{570 - 440}{440} \times 100$$

$$= 29.54\% \quad \textbf{Ans.}$$

In phasor diagram shown in Fig. 15.55 (*b*), drawn for 0.6 pf (leading)

$$OE = \text{Generated emf} = 388 \text{ V}$$

Excitation corresponding to 388 V, from OCC is 4.2 A

$$\text{So } OI_1 = 4.2 \text{ A}$$

$$I_1I_2 = 3 \text{ A}$$

$$\text{Total excitation } OI_2 = 3.2 \text{ A}$$

Induced emf corresponding to excitation of 3.2 A,

$$E_0 = 320 \text{ V}$$

$$\% \text{ Regulation} = \frac{320 - 440}{440} \times 100 = -27.27\% \quad \textbf{Ans.}$$

Figure 39 : M2-T2-Q3(Ans.)

Q4. Find the regulation by Zero Power Factor method of 5000kVA, 6600V, 3Φ, 50Hz star-connected alternator at full load, unity power factor having the following test data

Field Current(A)	32	50	75	100	140
OC terminal voltage(V)	3100	4900	6600	7500	8300
Full load current zero pf tests line pd(V)	0	1850	4250	5800	7000

Neglect armature resistance.

Solution: From the above datas OCC and full-load zero power factor curves are drawn, as shown in Fig. 15.58.

From the Potier ΔBDF

Field current required to overcome demagnetising effect of armature reaction on full load, BF = 23 amperes

Full load reactance drop, DF = 900 volts

In phasor diagram,

OV = Terminal line voltage of 6,600 volts

VE = 900 volts representing line reactance drop

$$\therefore \ \mathrm{OE} = \sqrt{(\mathrm{OV})^2 + (\mathrm{VE})^2 - 2 \times \mathrm{OV} \times \mathrm{VE} \cos(90^\circ + \phi)}$$

$$= \sqrt{(6{,}600)^2 + (900)^2 - 2 \times 6{,}600 \times 900 \cos 90^\circ} = 6{,}660 \text{ V}$$

As seen from OCC field current required to induce 6,660 line volts is 76 amperes. Hence phasor $\mathrm{OI_1} = 76$ A and is perpendicular to phasor OE and phasor $\mathrm{I_1 I_2} = 23$ A and is parallel to phasor OI.

Figure 40 : M2-T2-Q4(Ans.)

Total field current, $\mathrm{OI_2} = 79.6$ A

From OCC it is found that line emf corresponding to this field current of 79.6 A is 6,820 volts.

$$\therefore \ E_0 = 6{,}820 \text{ volts}$$

$$\text{Percentage regulation} = \frac{E_0 - V}{V} \times 100 = \frac{6{,}820 - 6{,}600}{6{,}600} \times 100$$

$$= 3.33\ \%\ \textbf{Ans.}$$

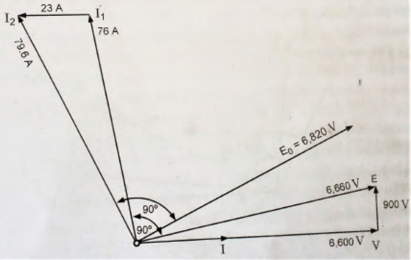


Figure 41 : M2-T2-Q4(Ans.)

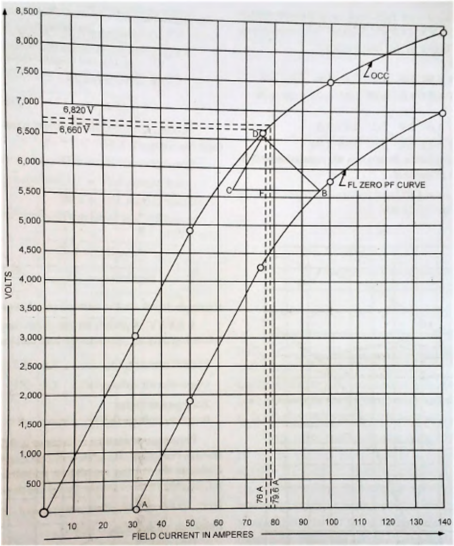


Figure 42 : M2-T2-Q4(Ans.)

Q5. A 3.5MVA star connected alternator rated 4160V at 50Hz has open-circuit characteristics given by the following data

Field Current(A)	50	100	150	200	250	300	350	400
Line emf (V)	1620	3150	4160	4750	5130	5370	5550	5650

A field current of 200A is found necessary to circulate full load current on short circuit of alternator. Calculate the full load voltage regulation at 0.8pf lagging by ampere-turn method.

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Thank You

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Synchronous and Induction Machines

Synchronous and Induction Machines (EE-202)

by

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Asst. Professor
Dept. of EEE, MACE

1. Theory of Salient Pole Machine
    - Blondel's Two Reaction Theory
    - Slip test (Determination of X_d & X_q)
2. Parallel Operation of Alternators
    - Requirements for Parallel Operation
    - Methods of Synchronisation
        - Dark Lamp Method
        - Bright Lamp Method
        - Synchroscope
    - Effects of Changing Excitation
    - Load Sharing between Two Alternators

- **Cylindrical rotor** $\rightarrow$ uniform air-gap $\rightarrow$ same reactance irrespective of the spatial position of rotor $\rightarrow$ Synchronous reactance,X_s(constant for all positions of field poles w.r.t. armature.)

1. Theory of Salient Pole Machine

- **Cylindrical rotor** → uniform air-gap → same reactance irrespective of the spatial position of rotor → Synchronous reactance,X_s(constant for all positions of field poles w.r.t. armature.)
- **Salient pole machine** → non-uniform air-gap → reactance varies with rotor position.

- **Cylindrical rotor** → uniform air-gap → same reactance irrespective of the spatial position of rotor → Synchronous reactance,X_s(constant for all positions of field poles w.r.t. armature.)
- **Salient pole machine** → non-uniform air-gap → reactance varies with rotor position.
- Two axes of geometry
  1. **Direct Axis** - Field pole axis
  2. **Quadrature Axis** - axis through the centre of inter-polar space

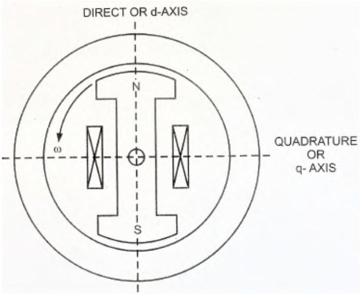

Figure 1 : Salient pole machine

- Reluctance of magnetic path $\rightarrow$ different along direct and quadrature axes.
- Reluctance of direct axis magnetic path is due to

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- Reluctance of quadrature axis magnetic path is mainly due to $\rightarrow$

1. Theory of Salient Pole Machine

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- Reluctance of direct axis magnetic path is due to → yoke, teeth of stator, air-gap, pole, core of rotor etc.
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- Reluctance of magnetic path $\rightarrow$ different along direct and quadrature axes.
- Reluctance of direct axis magnetic path is due to $\rightarrow$ yoke, teeth of stator, air-gap, pole, core of rotor etc.
- Reluctance of quadrature axis magnetic path is mainly due to $\rightarrow$ large air-gap in inter-polar space
- Due to non-uniformity of reluctance of magnetic paths, armature mmf has two components
  1. Direct acting component
  2. Quadrature component

Armature reaction:

- Unity pf $\rightarrow$

1. Theory of Salient Pole Machine

Armature reaction:

- Unity pf $\rightarrow$ cross magnetising or distorting effect $\rightarrow$

Armature reaction:

- Unity pf $\rightarrow$ cross magnetising or distorting effect $\rightarrow$ armature mmf acts at right angles to the axis of salient pole

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- ZPF lagging $\rightarrow$

1. Theory of Salient Pole Machine

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- Unity pf $\rightarrow$ cross magnetising or distorting effect $\rightarrow$ armature mmf acts at right angles to the axis of salient pole
- ZPF lagging $\rightarrow$ demagnetizing effect $\rightarrow$

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- ZPF lagging $\rightarrow$ demagnetizing effect $\rightarrow$ armature mmf acts directly upon magnetic path though salient pole $\rightarrow$ directly opposing
- ZPF leading $\rightarrow$

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- ZPF lagging $\rightarrow$ demagnetizing effect $\rightarrow$ armature mmf acts directly upon magnetic path though salient pole $\rightarrow$ directly opposing
- ZPF leading $\rightarrow$ magnetizing effect $\rightarrow$ armature mmf acts directly upon magnetic path though salient pole $\rightarrow$ directly aiding

In general, if $0 < \theta < 90$

- armature mmf has both direct acting and quadrature component.
- Direct component $\propto I_d \propto Sin\theta$
- Quadrature component $\propto I_q \propto Cos\theta$
  where $\theta =$ angle between armature current and excitation voltage(E_o)

- Two reactance concept and synchronous impedance concept
- **Synchronous impedance concept** $\rightarrow$ Effect of armature reaction is taken into account by means of equivalent armature reactance voltage.

1. Theory of Salient Pole Machine

- Two reactance concept and synchronous impedance concept
- **Synchronous impedance concept** $\rightarrow$ Effect of armature reaction is taken into account by means of equivalent armature reactance voltage.
- Due to the difference in reluctance of magnetic paths, **two reactance concept** replaces effect of armature reaction by two fictitious voltages.
- Reactance voltages are $I_d X_{ad}$ and $I_q X_{aq}$

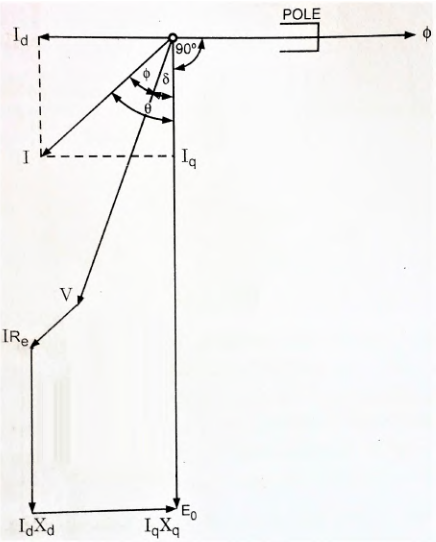

Figure 2 : Phasor diagram

Assuming same armature leakage flux along direct and quadrature axes,

- Direct axis synchronous reactance, $X_d = X_{ad} + X_L$
- Quadrature axis synchronous reactance, $X_q = X_{aq} + X_L$

- Two reaction theory proposed by **Blondel**.
- Owing to the difference in magnetic paths,
  - Armature current can be resolved into two components, I_d and I_q
    - $I_d \perp E_o$
    - I_q along E_o
  - Armature reactance has two components
    - Direct axis armature reactance(X_{ad}) associated with I_d
    - Quadrature axis armature reactance(X_{aq}) associated with I_q

- Voltage equation for each phase based on two-reactance concept

$$\vec{V} = \vec{E_o} - \vec{I} R_e - \vec{I_d} X_d - \vec{I_q} X_q$$

- Salient pole machine, $X_q = 0.6$ to 0.7 times X_d
- Cylindrical rotor machine, $X_q = X_d$

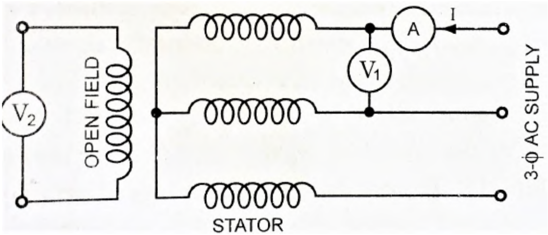

Figure 3 : Slip Test Circuit

- Apply a balanced reduced external voltage(V_1) to an unexcited machine at low speed little less than N_s(Slip $< 1\%$)
- Applied voltage(V_1), armature current(I) and induced voltage in the field(V_2) are measured by oscillographs.

- $V_1 \rightarrow I \rightarrow$ stator mmf
- stator mmf moves slowly relative to poles and induces emf in the field winding
- The physical poles and armature reaction mmf are alternatively in-phase and out, change occurring at slip frequency
- When axis of poles and axis of armature reaction mmf wave coincides, armature mmf acts through field magnetic circuit. $\Longrightarrow$ Applied voltage will be equal to the drop caused by direct axis component of armature reaction and leakage reactance.
- When armature reaction mmf is in quadrature with the field poles, applied voltage is equal to drop due to cross magnetising component of armature reaction and leakage reactance.

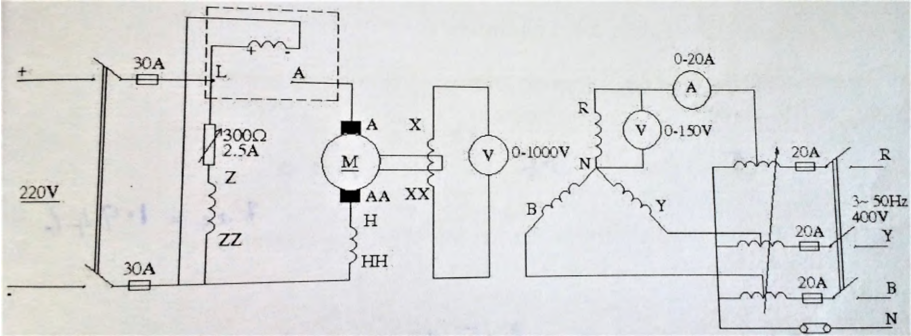

Figure 4 : Connection diagram - Slip Test

1.2 Slip test (Determination of X_d & X_q)

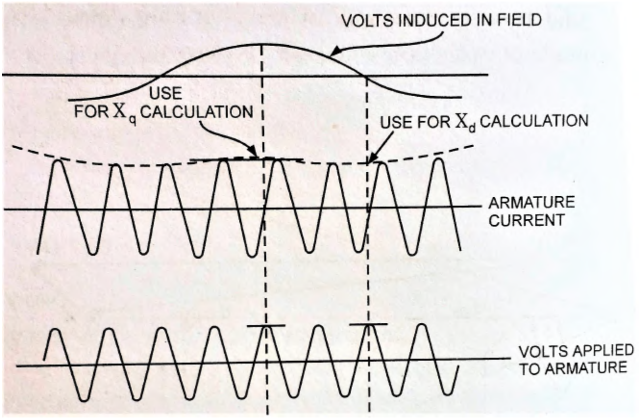

Figure 5 : Slip Test - waveforms

$$X_d = \frac{Maximum\ voltage}{Minimum\ current}, \qquad X_q = \frac{Minimum\ voltage}{Maximum\ current}$$

1.2 Slip test (Determination of X_d & X_q)

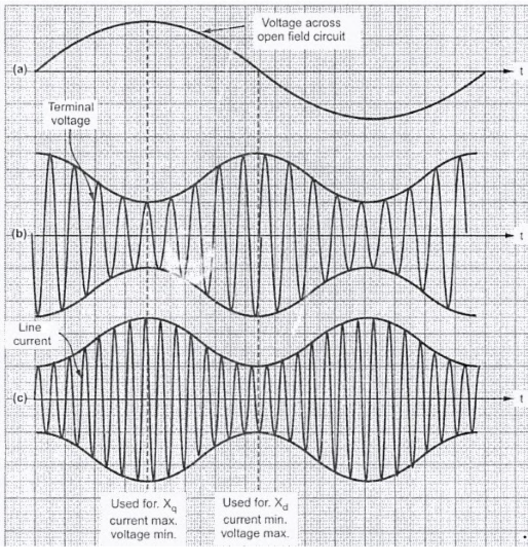

Figure 6 : Slip Test - waveforms

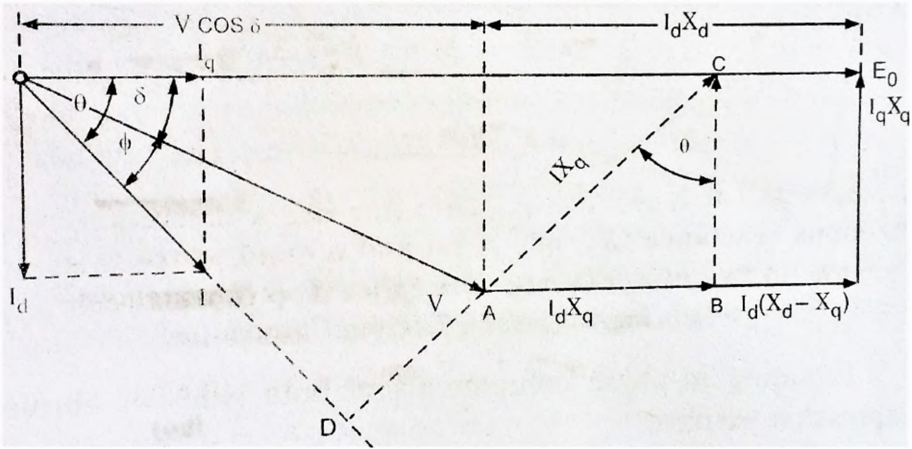

Figure 7 : Phasor diagram

- Power developed per phase(P_d) = Power output(P_{out}) per phase

- Power developed per phase(P_d) = Power output(P_{out}) per phase (provided R is negligible)

$$
\begin{aligned}
P_d &= P_{out} = VICos\Phi \\
I_q X_q &= VSin\delta \\
I_d X_d &= E_o - VCos\delta \\
ICos\Phi &= I_d Sin\delta + I_q Cos\delta \\
P_d &= VI_d Sin\delta + VI_q Cos\delta \\
&= \frac{E_o V}{X_d} Sin\delta + \frac{V^2}{2}\left[\frac{1}{X_q} - \frac{1}{X_d}\right] Sin2\delta
\end{aligned}
$$

Power Developed by an Alternator

- Power developed per phase(P_d) = Power output(P_{out}) per phase (provided R is negligible)

$$
\begin{aligned}
P_d &= P_{out} = VICos\Phi \\
I_q X_q &= VSin\delta \\
I_d X_d &= E_o - VCos\delta \\
ICos\Phi &= I_d Sin\delta + I_q Cos\delta \\
P_d &= VI_d Sin\delta + VI_q Cos\delta \\
&= \frac{E_o V}{X_d} Sin\delta + \frac{V^2}{2}\left[\frac{1}{X_q} - \frac{1}{X_d}\right] Sin2\delta
\end{aligned}
$$

- Total power developed

Power Developed by an Alternator

- Power developed per phase(P_d) = Power output(P_{out}) per phase (provided R is negligible)

$$
\begin{aligned}
P_d &= P_{out} = VICos\Phi \\
I_qX_q &= VSin\delta \\
I_dX_d &= E_o - VCos\delta \\
ICos\Phi &= I_dSin\delta + I_qCos\delta \\
P_d &= VI_dSin\delta + VI_qCos\delta \\
&= \frac{E_oV}{X_d}Sin\delta + \frac{V^2}{2}\left[\frac{1}{X_q} - \frac{1}{X_d}\right]Sin2\delta
\end{aligned}
$$

- Total power developed

$$P = \frac{3E_oV}{X_d}Sin\delta + \frac{3V^2}{2}\left[\frac{1}{X_q} - \frac{1}{X_d}\right]Sin2\delta$$

- First term $\rightarrow$ power due to field excitation
- Second term $\rightarrow$ reluctance power(power due to saliency)
- For cylindrical machine, power due to saliency $= 0$ ($\because X_d = X_q$)
- For Alternator, $\delta = +$ve
- For synchronous motor, $\delta =$ -ve

Q1. A star connected salient pole alternator is driven at a speed near synchronous speed with field circuit open and stator is supplied from a three phase supply. Voltmeter connected across the line gave minimum and maximum readings of 2800V and 2820V. The line current fluctuated between 360A and 275A. Find X_d and X_q per phase. Neglect R_e.

Ans:

$$X_d = \frac{Maximum\ voltage}{Minimum\ current}, \quad X_q = \frac{Minimum\ voltage}{Maximum\ current}$$

$$X_{d-line} = \frac{2820}{275} = 10.25\Omega, \quad X_{d-phase} = \frac{10.25}{\sqrt{3}} = 5.92\Omega$$

$$X_{q-line} = \frac{2800}{360} = 7.78\Omega, \quad X_{q-phase} = \frac{7.78}{\sqrt{3}} = 4.5\Omega$$

Q2. A 2.2kV, 50Hz, three phase star connected alternator has $R_e = 0.5\Omega$ per phase. A field current of 30A produced full load current of 200A on short-circuit test and line to line emf of 1.1kV on open circuit test. Determine (a) power angle of alternator when it delivers full load at 0.8pf lag (b) Short circuit ratio(SCR) of the alternator.

Ans:

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{2200}{\sqrt{3}} = 1270.2V$$

$$Full\ load\ current, I = 200A$$

$$Synchronous\ impedance\ per\ phase, Z_s = \frac{E_{o-phase}}{I_{sc-phase}} = \frac{(1100/\sqrt{3})}{200} = 3.175\Omega$$

$$Synchronous\ reactance\ per\ phase, X_s = \sqrt{Z_s^2 - R_e^2}$$

$$= \sqrt{3.175^2 - 0.5^2} = 3.136\Omega$$

$$When\ delivering\ full\ load\ current,$$

$$Open\ circuit\ voltage\ per\ phase, E = \sqrt{(VCos\Phi + IR_e)^2 + (VSin\Phi + IX_s)^2}$$

$$= \sqrt{(1270.2 \times 0.8 + 200 \times 0.5)^2 + (1270.2 \times 0.6 + 200 \times 3.136)^2} = 1782V$$

$$Total\ power\ output = 3V_{ph}I_{ph}Cos\Phi = 3 \times \frac{2200}{\sqrt{3}} \times 200 \times 0.8 = 609681W$$

$$(Assuming\ non-salient\ pole\ alternator\ and\ neglecting\ losses)$$

$$Total\ power\ developed = \frac{3E_oV}{X_s}Sin\delta$$

$$\implies Sin\delta = \frac{PX_s}{E_oV} = \frac{609681 \times 3.136}{3 \times 1782 \times 1270.2} = 0.2816$$

$$\therefore Power\ angle, \delta = Sin^{-1}(0.2816) = 16.35^o$$

$$SCR = \frac{1}{X_s} = \frac{1}{3.316} = 0.319$$

- Efficiency

2. Parallel Operation of Alternators

- Efficiency
- Reliability or Continuity of Service

- Efficiency
- Reliability or Continuity of Service
- Maintenance and Repair

2. Parallel Operation of Alternators

- Efficiency
- Reliability or Continuity of Service
- Maintenance and Repair
- Physical Size

2.1 Requirements for Parallel Operation

- Alternators must have same output voltage rating
- Speeds of the machines should be such as to generate same frequency
- Alternators should be of same type so as to generate voltages of same waveform
- Prime movers of the alternators should have same speed-load characteristics
- Alternators should have reactances in their armature

Conditions for Synchronising

* **Synchronising** - The process of connecting an alternator in parallel with another alternator or with the common bus bar.
* **Running machine** - Alternators which are in operation and sharing the load
* **Incoming machine** - Alternator which is to be connected in parallel to the running machines.

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- The speed of incoming machine must be such that its frequency $\left(f = \frac{PN}{120}\right)$ equals to bus bar frequency
- Phase of incoming machine voltage must be the same as that of the bus bar voltage relative to the load.
- For three phase alternator, the phase sequence of incoming machine must be same as that of bus bar.

- Synchronising of single phase alternators
  1. Dark Lamp Method
  2. Bright Lamp Method
- Synchronising of three phase alternators
  1. Three Dark Lamp Method
  2. Two Bright and One Dark Lamp Method
  3. Synchroscope

Dark Lamp Method

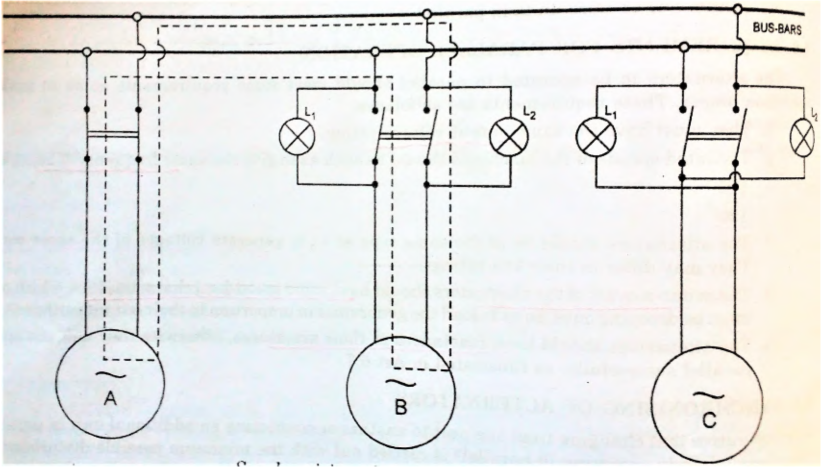

Figure 8 : Synchronising of single phase alternators

Dark Lamp Method

- Equality of terminal voltages can be determined by connecting voltmeters to the incoming and running alternators.
- Equality of frequency and phasing can be determined with the help of synchronising lamps.

Let alternator B is to be paralleled with alternator A,

- Prime mover of Alternator B is started and brought up close to the rated speed.
- Alternator B is excited and its voltage is raised by increasing the excitation.
- If frequencies of alternators A & B are exactly same and their terminal voltages w.r.t local series circuit $\rightarrow$ no resultant voltage will act across lamps L_1 and L_2 $\rightarrow$ Lamps will remain dark.
- If frequencies of A & B are not equal $\rightarrow$ flickering of lamps.
- Frequency of flickering = Difference of frequencies of two alternators. $\implies$ greater the difference in frequencies, greater will be the frequency of flickering.

Dark Lamp Method

- Synchronising is done at the middle of dark period.

"At the time of synchronising of alternators, the speed of the incoming machine is adjusted until the lamps go in and out very slowly, the terminal voltage of the incoming alternator is made equal to bus bar voltage by adjusting the excitation of the incoming alternator and then switch of incoming alternator is closed in the middle of the dark period."

- But it is not so easy to judge the middle of dark period.

Bright Lamp Method

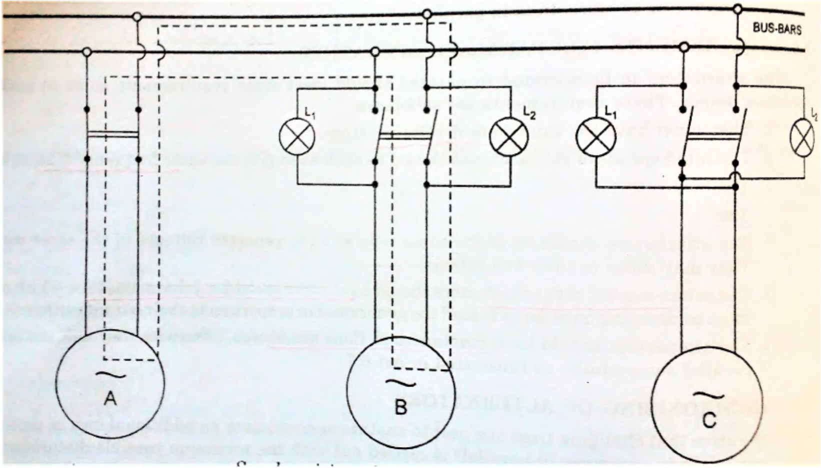

Figure 9 : Synchronising of single phase alternators

Bright Lamp Method

- Easy to judge the middle of bright period
- In **Bright lamp method**, lamps are cross-connected.
- Maximum voltage across lamps will occur when alternator C is in phase opposition with alternator A.
- Magnitude of maximum voltage that can exist across a lamp is double that of the voltage of one machine.
- After doing the adjustments, the synchronising switch is closed at the middle of bright period.

"Phasing out the alternator" $\rightarrow$ The process of checking the phase sequence and getting it correct

1. Three Dark Lamp Method
2. Two Bright and One Dark Lamp Method
3. Using Synchroscope

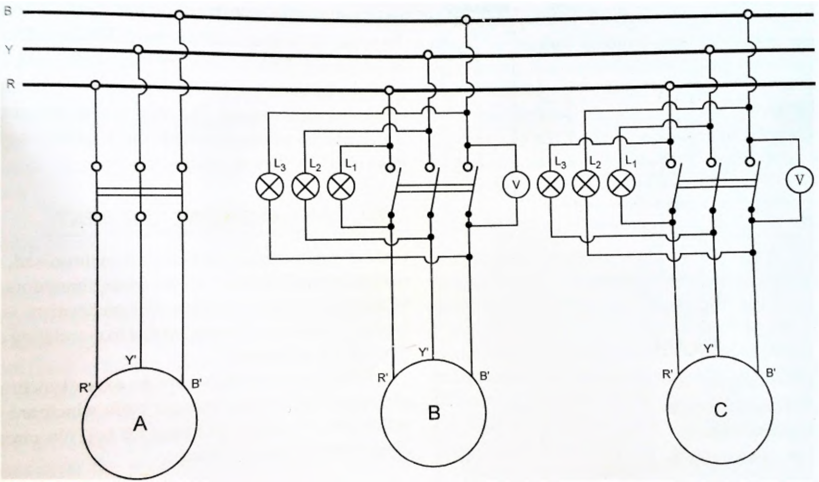

Figure 10 : Synchronising of 3-Phase Alternators

Three Dark Lamp Method

- Prime mover of alternator B is started and brought up close to the rated speed.
- Then alternator B is excited and its voltage is raised by increasing the excitation.
- If the incoming alternator B is properly connected, all the three lamps should become bright and dark together.
- If they bright and dim in sequence $\Longrightarrow$ incoming alternator B is not properly connected with the bus-bars and the phase sequence of incoming alternator B must be reversed relative to the system. (Interchange any two leads on either the alternator side or the line side of the switch)
- The speed of incoming machine is further adjusted until the lamps flicker at a very rate and voltage is made equal to the bus bar voltage by adjusting the excitation.
- The synchronising switch is closed at the instant all the three lamps are dark.

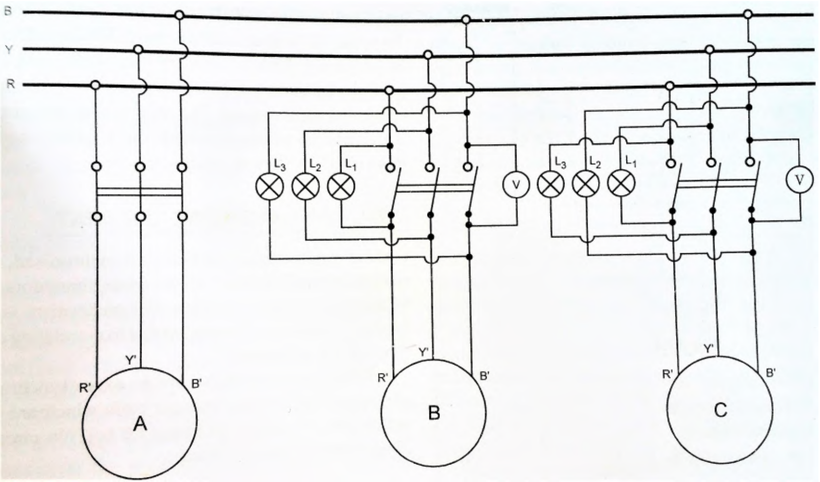

Figure 11 : Synchronising of 3-Phase Alternators

Two Bright and One Dark Lamp Method

- When incoming alternator is in synchronism with running machine, lamps L_1 and L_3 are bright and L_2 is dark.
- Since near the point of synchronism, the brightness of one lamp increases and of another decreases. The instant at which the incoming machine is in synchronism with the bus-bars can be accurately determined and the paralleling switch will be closed at this instant.
- If the incoming machine is too fast $\rightarrow$ voltage across L_3 decreases, L_1 increases and L_2 decreases.
- If the incoming machine is too slow $\rightarrow$ voltage across L_3 increases, L_1 decreases and L_2 increases.
- Hence when the three lamps are placed in a ring, a light wave travelling in counter-clockwise direction indicates that the incoming machine is slow and light wave travelling in clockwise direction indicates that the incoming machine is fast.
- Synchronising switch is closed when changes in light are very slow and at the instant the lamp L_2 is dark.

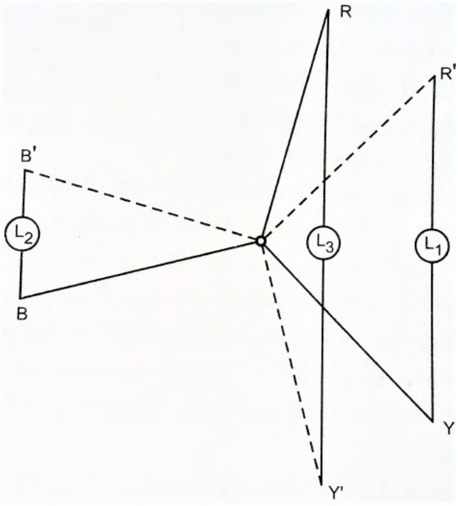

Figure 12 : Phasor

Using Three Limbed Transformer and Lamp

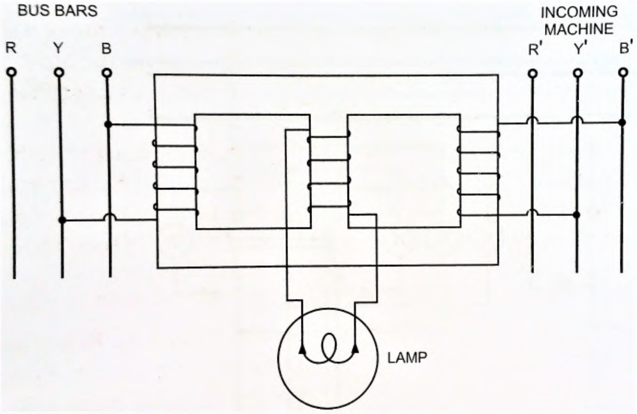

Figure 13 : Three Limbed Transformer and Lamp

Using Three Limbed Transformer and Lamp

- The two primary windings on the outer limbs are connected to bus-bar and incoming alternator.
- secondary winding on the central limb is connected to the lamp
- The correct moment for closing paralleling switch is the middle of dark period if primaries are each connected the same way round, and the middle of the bright period if the connections of one winding are reversed.

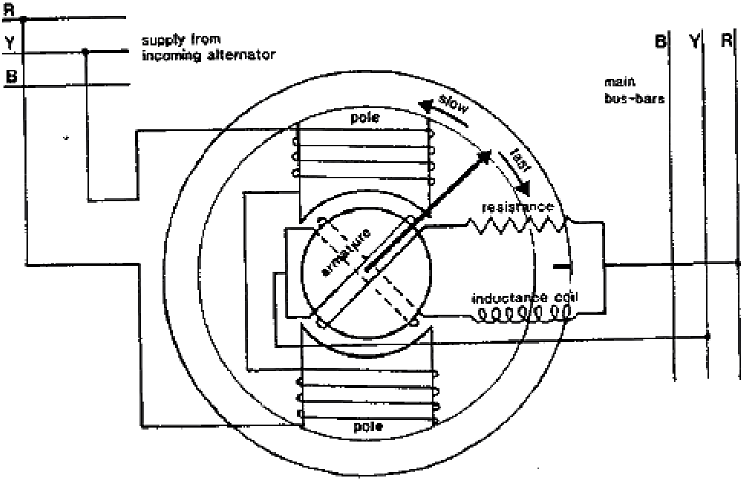

Figure 14 : Synchroscope

Synchroscope

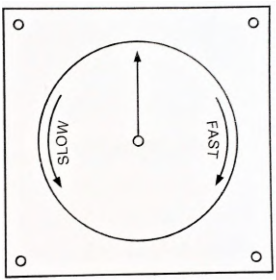

Figure 15 : Synchroscope-Dial

Synchroscope

- Synchroscope $\rightarrow$ Instrument indicating the difference of phase and frequency between two voltages.
- Its a split-phase motor
- Torque is developed if the frequencies of the two voltages differ.
- Voltages from corresponding phases of incoming and running alternators are applied to synchroscope.
- When frequencies are equal, no torque is exerted on the pointer and the pointer stops at the middle of the dial.
- When pointer stops at vertical position, the frequencies are equal, the voltages are in phase and the paralleling switch can be closed.

Synchronising Current

- Once synchronisation is done, the machine will try to remain in synchronism with other alternators.
- Any tendency to depart from synchronism is opposed by synchronising torque produced due to circulating current flowing through the alternators.

Synchronising Current

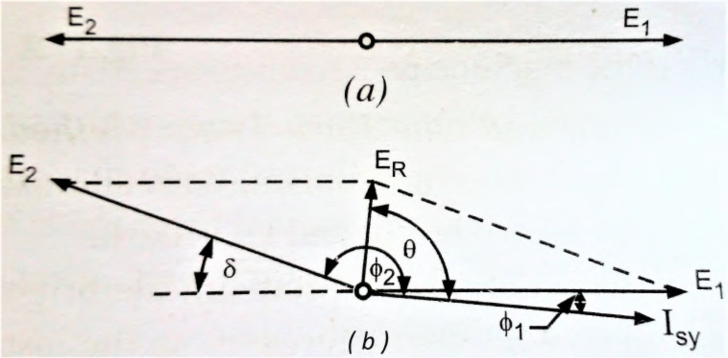

Figure 16 : Voltage Phasors

- When two alternators are in synchronism $\rightarrow$ have equal emfs in exact phase opposition $\rightarrow$ no circulating current (Fig. (a))

- When two alternators are in synchronism $\rightarrow$ have equal emfs in exact phase opposition $\rightarrow$ no circulating current (Fig. (a))
- When induced emfs are equal in magnitude but not in exact phase opposition $\rightarrow$ resultant emf around the local circuit $\neq$ zero $\rightarrow$ causes flow of current $\rightarrow$ **Synchronising current(I_{sy})**

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- When induced emfs are equal in magnitude but not in exact phase opposition $\rightarrow$ resultant emf around the local circuit $\neq$ zero $\rightarrow$ causes flow of current $\rightarrow$ **Synchronising current(I_{sy})**

$\rightarrow$ Let alternator-2 tends to retard,

- E_2 falls back by a phase angle δ electrical degrees
- $|E_1| = |E_2| =$ E
  but phase difference = 180 - δ

→ Resultant emf(E_R)

$$E_R = 2ECos\left(\frac{180-\delta}{2}\right)$$
$$= 2ECos\left(90-\frac{\delta}{2}\right)$$
$$= 2ESin\frac{\delta}{2}$$
$$= 2E \times \frac{\delta}{2}$$
$$= E\delta$$

→ Resultant emf(E_R)

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Synchronising Current

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$$= 2E \times \frac{\delta}{2}$$
$$= E\delta$$

→ Synchronising current(I_{sy})

$$I_{sy} = \frac{E_R}{Z_{cs}} = \frac{E\delta}{Z_{cs}}$$

where Z_{cs}=combined synchronous impedance/phase of the 2 alternators

- I_{sy} lags behind E_R by an angle θ

Synchronising Current

- I_{sy} lags behind E_R by an angle θ
- If $R_{ce} <<< X_{cs}$,

$$I_{sy} = \frac{E_R}{X_{cs}}$$

$\implies I_{sy}$ lags E_R by $90^o \implies$ almost in phase with E_1

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- If $R_{ce} <<< X_{cs}$,

$$I_{sy} = \frac{E_R}{X_{cs}}$$

$\implies$ I_{sy} lags E_R by 90^o $\implies$ almost in phase with E_1

- $I_s y$ $\rightarrow$ generating current w.r.t machine 1 and motoring current w.r.t machine 2
- I_{sy} sets up synchronising torque(T_{sy}) which tends to accelerate machine no.2 and decelerate machine no.1
- Any departure from synchronism results in development of synchronising torque which tends to keep the machines in synchronism

- Power supplied by Machine no.1 =

Synchronising Power

- Power supplied by Machine no.1 $= E_1 I_{sy} Cos\Phi_1$

Synchronising Power

- Power supplied by Machine no.1 $= E_1 I_{sy} Cos\Phi_1$
- Power received by Machine no.2 $=$

Synchronising Power

- Power supplied by Machine no.1 $= E_1 I_{sy} Cos\Phi_1$
- Power received by Machine no.2 $= E_2 I_{sy} Cos\left(180 - \Phi_2\right)$

Synchronising Power

- Power supplied by Machine no.1 $= E_1 I_{sy} Cos\Phi_1$
- Power received by Machine no.2 $= E_2 I_{sy} Cos\,(180 - \Phi_2)$
- Power supplied by Machine no.1 = Power received by Machine no.2 + copper losses

Synchronising Power

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- Power received by Machine no.2 $= E_2 I_{sy} Cos\,(180 - \Phi_2)$
- Power supplied by Machine no.1 = Power received by Machine no.2 + copper losses
- Power supplied by Machine no.1 is called as **Synchronising power(P_{sy})**

Synchronising Power

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- Power received by Machine no.2 $= E_2 I_{sy} Cos\left(180 - \Phi_2\right)$
- Power supplied by Machine no.1 = Power received by Machine no.2 + copper losses
- Power supplied by Machine no.1 is called as **Synchronising power(P_{sy})**

$$\begin{aligned} P_{sy} &= E_1 I_{sy} Cos\Phi_1 \\ &= E_1 I_{sy} \\ &= E \times \frac{E\delta}{X_{cs}} \\ &= \frac{\delta E^2}{X_{cs}} \end{aligned}$$

- Total synchronising power =

Synchronising Power

- Power supplied by Machine no.1 $= E_1 I_{sy} Cos\Phi_1$
- Power received by Machine no.2 $= E_2 I_{sy} Cos\left(180 - \Phi_2\right)$
- Power supplied by Machine no.1 = Power received by Machine no.2 + copper losses
- Power supplied by Machine no.1 is called as **Synchronising power(P_{sy})**

$$
\begin{aligned}
P_{sy} &= E_1 I_{sy} Cos\Phi_1 \\
&= E_1 I_{sy} \\
&= E \times \frac{E\delta}{X_{cs}} \\
&= \frac{\delta E^2}{X_{cs}}
\end{aligned}
$$

- Total synchronising power $= 3P_{sy} = \frac{3\delta E^2}{X_{cs}}$

Synchronising Torque

Synchronising Torque

$$3P_{sy} = T_{sy} \times \frac{2\pi N_s}{60}$$

Therefore Synchronising torque,

$$T_{sy} = \frac{3P_{sy} \times 60}{2\pi N_s}$$

where N_s is the synchronous speed in rpm

Synchronising Torque

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Therefore Synchronising torque,

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where N_s is the synchronous speed in rpm

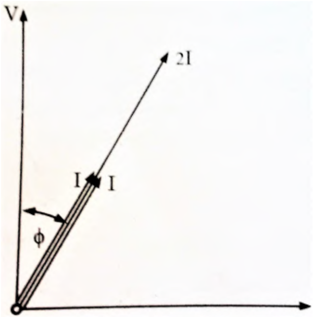

Figure 17 : (a)

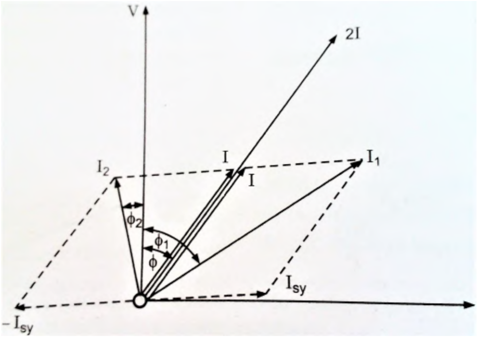

Figure 18 : (b)

2.3 Effects of Changing Excitation

- Let two identical alternators are sharing equally a load of pf $Cos\Phi$
  - If both the machines have same excitation, $\implies |I_1| = |I_2| = \mathrm{I}$
    Also I_1 & I_2 are in phase(Fig. (a))
- If the excitation of alternator 1 is increased,(Fig. (b))
  - $|E_1| > |E_2| \implies I_{sy}$ flows
  - I_{sy} is almost in quadrature with V
  - Output current of alternator 1,
    $\vec{I_1} = \vec{I} + \vec{I_{sy}}$
  - Output current of alternator 2,
    $\vec{I_2} = \vec{I}$ - $\vec{I_{sy}}$
  - For alternator 1, $\Phi_1 > \Phi \implies Cos\Phi_1 < Cos\Phi \implies$ pf decreases.
  - For alternator 2, $\Phi_2 < \Phi \implies Cos\Phi_2 > Cos\Phi \implies$ pf improves.
  - **By changing the excitation, the power factors of alternators are changed.**

2.3 Effects of Changing Excitation

- I_{sy} doesn't change wattful(active) components but changes the wattless(reactive) components.
- Due to change in excitation, the output current of alternators changes with no appreciable change in it's active power(kW).

- During parallel operation of two alternators, increase in excitation of alternator 1 causes,

- increase in terminal voltage of alternator 1
- increase in reactive power supplied by alternator 1
- decrease in reactive power supplied by alternator 2

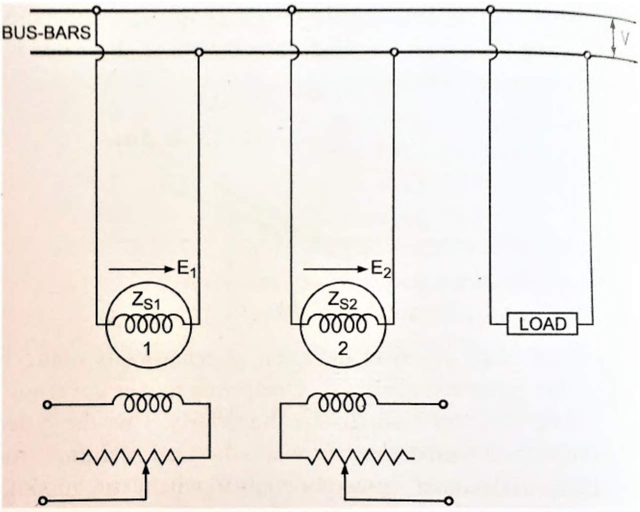

Figure 19 : Load sharing

2.4 Load Sharing between Two Alternators

- Consider two Alternators(A_1 and A_2) with identical speed-load characteristics running in parallel.

Let,

- V = Common terminal voltage in volts
- Z = Load impedance
- E_1 = Generated emf of A_1
- E_2 = Generated emf of A_2
- Z_{s1} = Synchronous impedance per phase of A_1
- Z_{s2} = Synchronous impedance per phase of A_2

- Terminal voltage of A_1

$$
\begin{aligned}
V &= E_1 - I_1 Z_{s1} \\
\implies I_1 &= \frac{E_1 - V}{Z_{s1}}
\end{aligned}
$$

- Terminal voltage of A_2

$$
\begin{aligned}
V &= E_2 - I_2 Z_{s2} \\
\implies I_2 &= \frac{E_2 - V}{Z_{s2}}
\end{aligned}
$$

Therefore,

$$
I_1 + I_2 = \frac{E_1 - V}{Z_{s1}} + \frac{E_2 - V}{Z_{s2}}
$$

- Voltage across load

$$
\begin{aligned}
V &= IZ = (I_1 + I_2)Z \\
\implies \frac{V}{Z} &= \frac{E_1 - V}{Z_{s1}} + \frac{E_2 - V}{Z_{s2}} \\
\implies V\left(\frac{1}{Z_{s1}} + \frac{1}{Z_{s2}} + \frac{1}{Z}\right) &= \frac{E_1}{Z_{s1}} + \frac{E_2}{Z_{s2}} \\
\implies V &= \frac{\left(\frac{E_1}{Z_{s1}} + \frac{E_2}{Z_{s2}}\right)}{\left(\frac{1}{Z_{s1}} + \frac{1}{Z_{s2}} + \frac{1}{Z}\right)}
\end{aligned}
$$

Q1. Two single phase alternators with emfs $E_1 = 100V$ & $E_2 = 110V$ and synchronous impedances $Z_{s1} = (0.2 + j1)\,\Omega$ & $Z_{s2} = (0.2 + j1)\,\Omega$ operate in parallel on a load impedance of $Z = (3 + j4)\,\Omega$. Determine the terminal voltage and power output of each machine.

Ans:
V $=$ 96 - j3.87V
I_1 $=$ 5.457∠-34.64A
I_2 $=$ 14.24∠-63.24A
P_1 $=$ 443W
P_2 $=$ 664.8W

Q2. Two alternators running in parallel supply a lighting load of 2000kW and a motor load of 4000kW at pf 0.8 lagging. One machine is loaded to 2400kW at pf 0.95 lagging. What is the output and power factor of the second machine.

Ans:
$P_2 = 3600\text{kW}$
$Q_2 = 2211.16\text{kVAR}$
$pf_2 = 0.8521$ lagging

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Thank You

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Synchronous and Induction Machines

Synchronous and Induction Machines (EE-202)

by

Prof. Dinto Mathew
Asst. Professor
Dept. of EEE, MACE

Module 4 - Overview

1.1 Construction

- Synchronous motor → Synchronous machine operated as motor.
- Stator → Armature → Three-phase winding → connected to 3-phase AC supply
- Rotor → Field system → connected to DC supply.
- Can have either revolving armature or revolving field type construction. But revolving field type(ie, stationary armature type) is more common.
- Rating : 150rpm to 1500rpm, 150kW to 15MW.

Parts:

- **Laminated stator core & Stator winding(3-phase armature winding)**
    - Stator core & windings are similar to alternator.

- **Rotating field structure with damper winding & slip rings**
  - Salient pole field poles are commonly used.
  - Field construction is little different from alternator.
  - In addition to the field winding, a squirrel cage winding is also provided in the pole faces.
  - The slots on the rotor pole face are parallel to shaft.
  - The ends of this squirrel cage winding on the pole face are short circuited using copper bars.
  - This winding is known as **damper winding** or **amortisseur winding** $\rightarrow$ **starting winding**.

1. Synchronous motor

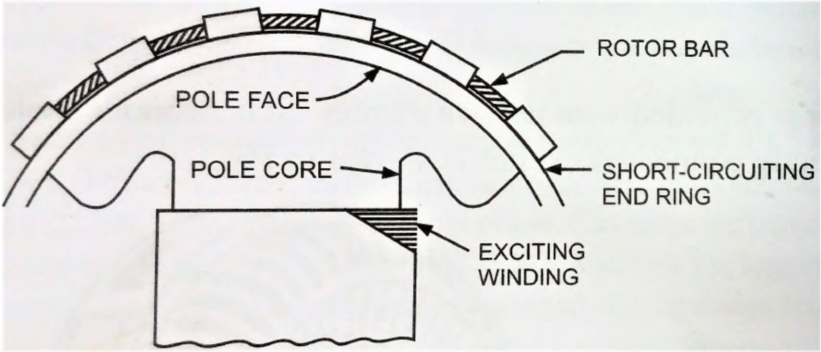


Figure 1 : Rotor pole

- **Brushes & brush holders**
- **End shields & shaft**

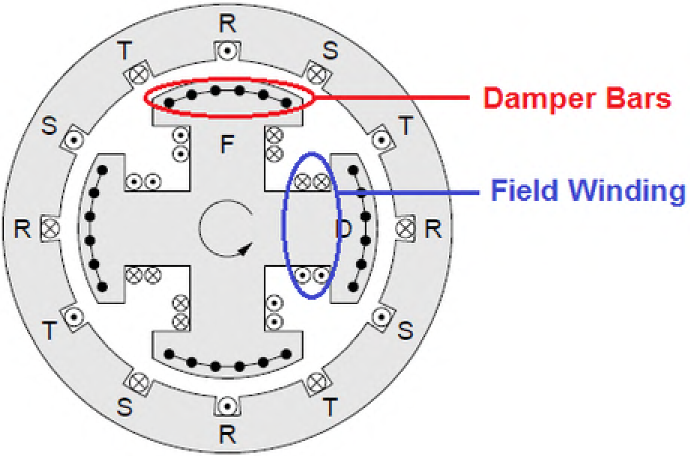


Figure 2 : Rotor pole with damper winding

- Field winding(rotor) is excited with DC supply $\rightarrow$ rotor poles are created $\rightarrow$ rotor poles retain the same polarity throughout.
- Stator winding is connected to AC supply $\rightarrow$ stator poles are created $\rightarrow$ polarity of stator poles changes.

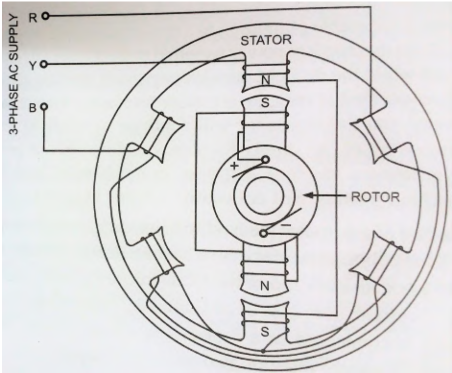


Figure 3 : Synchronous motor

1.2 Principle of Operation

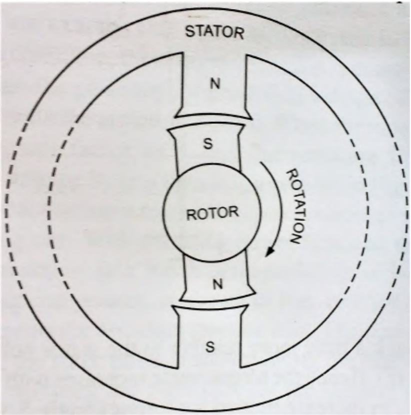


Figure 4 : Operation of synchronous motor(a)

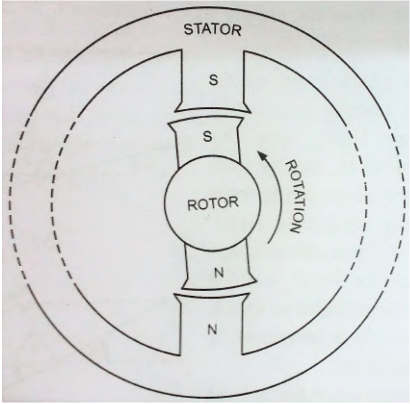


Figure 5 : Operation of synchronous motor(b)

- Consider stationary rotor in figure 4. Rotor S-pole is attracted to stator N-pole $\rightarrow$ rotor tends to rotate in clockwise direction.
- After half period(ie, $T/2$ seconds where $T=1/f$), polarity of stator poles is reversed. But polarity of rotor poles remains the same(figure 5).
- Rotor S-pole is repelled by stator S-pole $\rightarrow$ rotor tends to rotate in counter clockwise direction
- $\implies$ Torque acting on the rotor is not unidirectional but pulsating. Due to the moment of inertia of rotor, it will not move in any direction.
- **Synchronous motor has no self-starting torque**.

- Consider the rotor rotating(by some external means) in clockwise direction(figure 4).
- Rotor S-pole attracted by stator N-pole → clockwise torque
- After half period, stator poles polarity is reversed.
- If the rotor is rotated at such a speed by some external means at the starting moment that rotor S-pole advances by a pole pitch so that it is again under the influence of stator N-pole (figure 6).

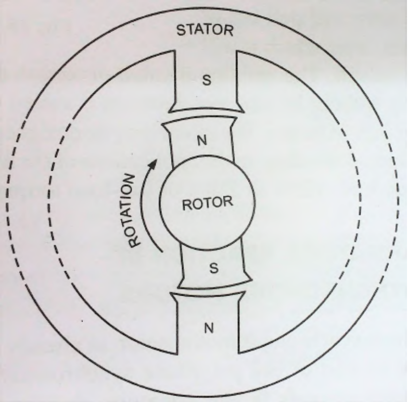


Figure 6 : Operation of synchronous motor(c)

- Torque acting on the rotor would be again clockwise $\rightarrow$ unidirectional torque.
- Now if the external means is removed, rotor will continue to rotate in clockwise direction under the influence of clockwise continuous torque acting on the rotor.
- For continuous torque, rotor must rotate at such a speed that it moves through the distance equal to pole pitch in half the period.(ie, in $\frac{T}{2}$ or $\frac{1}{2f}$ seconds).
- If there are 'P' number of poles, then the time taken for one complete revolution $= \frac{P}{2f}$ seconds
- ie, rotor should rotate at a speed $\frac{2f}{P} \times 60$ revolutions per minute.
- **For continuous torque, the rotor should rotate at synchronous speed($N_s = \frac{120f}{P}$)**

- Synchronous motor has **no self-starting torque**.
- Motor must be brought up to synchronous speed by some external means.

Methods

1. From DC source
2. By means of AC motor
3. By means of Damper winding

1.3 Starting - from DC source

- Synchronous motor is coupled with DC compound motor & started by means of DC compound motor.
- Speed of DC motor is adjusted by means of speed regulator.
- Synchronous motor is excited & synchronised with AC supply mains
- At the moment of synchronising, synchronous motor is switched on with the AC mains & DC motor is disconnected from the DC supply mains.
- Thereafter synchronous machine will operate as a motor & DC machine acts as a load on it.

1.3 Starting by means of AC motor

- Synchronous machine is coupled with induction motor
- Usually induction motor will have two poles less than the synchronous motor and hence induction motor will be capable of raising the speed of the synchronous machine to synchronous speed.
- Induction motor is normally started & the speed of synchronous machine is brought to synchronous speed.
- Synchronous machine must be synchronised with the bus-bars before switching the AC supply to synchronous motor.
- Thereafter synchronous machine will operate as a motor & induction machine can be uncoupled.

1.3 Starting by means of Damper winding

- **Synchronous motor is made self-starting by providing damper winding in the field pole faces.**
- Damper winding → short-circuited copper bars embedded in the faces of field poles
- Synchronous motor with damper winding starts as a squirrel cage induction motor
- AC supply is given to the stator winding → rotating magnetic field is produced → induces emf & hence currents in the damper winding → causes the rotor to rotate.
- When motor attains about 95% of synchronous speed, field winding(ie, rotor winding) is excited → rotor gets magnetically locked by the rotating magnetic field → machine will run as synchronous motor at synchronous speed.

1.3 Starting by means of Damper winding

Merits

- When machine is overloaded, it doesn't stop $\rightarrow$ continues running as a squirrel cage induction motor($\rightarrow$ When speed falls slightly below N_s due to overloading, an emf is induced in the damper winding which in turn produces torque and keeps the motor running even on overload)

Demerits

- Since damper winding resistance is low, at starting machine takes large current from the supply mains. To avoid large starting current, external resistance is added in the rotor winding during starting period.
- Field windings should be highly insulated for withstanding high induced emf due to induction at starting($\rightarrow$ Usually field windings are having low voltage ratings. eg: 110V or 250V).

Floating Condition

Figure 7 : Phasor during floating

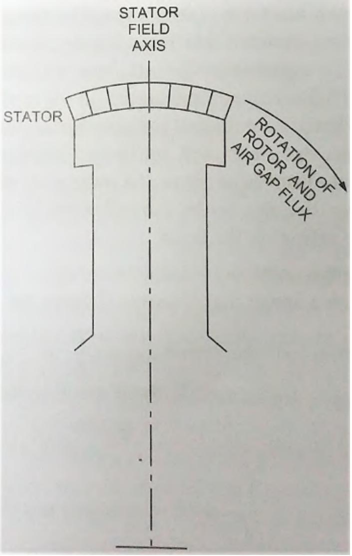


Figure 8 : Field alignment

Floating Condition

- When the synchronous machine(run by a prime mover) is synchronised, induced emf across the stator circuit of the motor(also called as **back emf** or **counter emf** or **excitation voltage**) is equal to the applied voltage 'V, but in opposite direction(fig. 7).
- The centre lines of stator pole & rotor DC field pole coincide(fig. 8).
- Since V = E, resultant voltage = 0 $\rightarrow$ so armature current = 0.
- Motor intake is zero as there is neither load nor losses to be met.
- This operating condition is known as **floating**.

Effect of Load on Synchronous Motor

No-load Condition

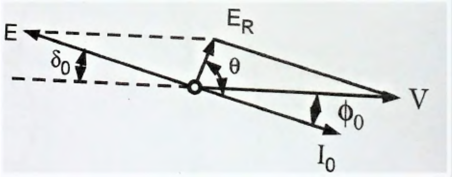


Figure 9 : Phasor at No-load Condition

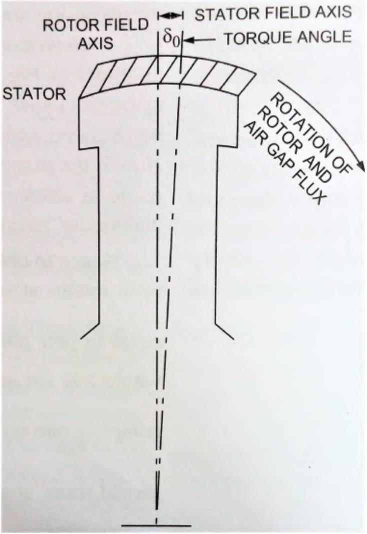


Figure 10 : Field alignment

No-load Condition

- After synchronising, when the external prime mover is removed, motor tends to slow down.
- There is no change in speed. But there is merely a shift in relative positions of stator & rotor poles & both of them continue to rotate at synchronous speed(N_s).
- Rotor poles falls back a little relative to the stator pole.
- Angular displacement between the rotor & stator poles is called as **torque angle(δ)**(fig. 10).
- Hereafter applied voltage(V) & induced emf(E) are no longer in phase opposition(E lags V by angle δ_o).
- Resultant of V & E, ie E_R causes a current(I_o) to flow in the stator winding.

$$I_o = \frac{E_R}{Z_s}$$

- I_o lags E_R by an angle called as **internal angle(θ)**(fig. 9).

Effect of Load on Synchronous Motor

- Internal angle$(\theta) = \tan^{-1}\left(\frac{X_s}{R_e}\right)$
  where $R_e =$ effective armature resistance per phase
  $X_s =$ synchronous reactance per phase $Z_s =$ synchronous impedance per phase
- At no-load, motor draws total power equal to $\sqrt{3}VI_o \cos\Phi_o$ from the AC supply.
- At no-load, torque angle(δ_o) is very small $\rightarrow$ resultant voltage(E_R) acting across the armature circuit is small $\rightarrow$ hence I_o is small.
- If we neglect motor losses at no-load, torque angle(δ_o) can be assumed as zero.

Effect of Load on Synchronous Motor

Loaded condition

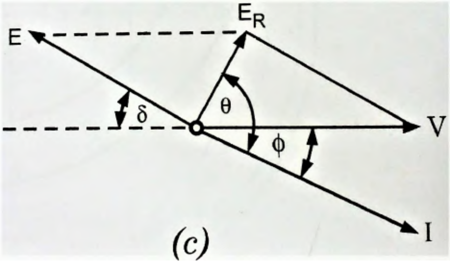


Figure 11 : Phasor at Loaded condition

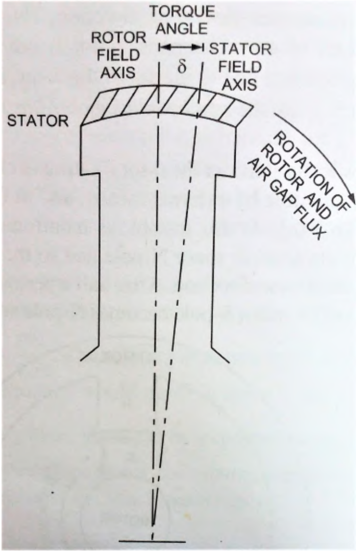


Figure 12 : Field alignment

Loaded condition

- When motor is loaded, rotor pole falls back a little more relative to the stator pole(fig. 12).
- Torque angle(δ_o) increases with increase in load.
- When motor is loaded, torque angle(δ_o) increases $\rightarrow$ resultant voltage(E_R) increases $\rightarrow$ current drawn from the supply(I) increases $\rightarrow$ input taken from the supply increases $\rightarrow$ output power increases $\rightarrow$ thus increased load is met(fig. 11).
- **Synchronous motor is able to supply increased mechanical load, not by reduction in speed, but by shift in relative positions of the rotor pole & stator pole**.
- If too great load is applied to synchronous motor, rotor will pull out of synchronism $\rightarrow$ thereafter motor will come to standstill.
- Maximum value of torque that the motor can develop without loosing synchronism is called **pull-out torque**.
- Usually $T_{pull-out} = (125\% \text{ to } 350\%)$ of T_{FL}.

- Change in field excitation doesn't affect speed or change in output. But does **affect the power factor & consequently the armature current** for a constant supply voltage & constant input power.
- Operation of synchronous motor at a constant supply needs nearly constant flux.
- DC source & AC source cooperate in establishing resultant constant flux.
- When DC excitation is weak $\rightarrow$ field produced by DC field will be weak $\rightarrow$ stator draws lagging current to magnetise the air gap to the extent needed to provide the necessary constant resultant flux.
- Drawing lagging current from the AC supply results in low lagging power factor.
- For constant applied voltage & constant input power, active component of current drawn from the supply should remain constant.

- With increase in DC excitation, magnetising component of current drawn from the supply is reduced → supply side power factor improves → results in reduction of armature current(ie, stator current I).
- If excitation is further increased such that power factor becomes unity, stator draws only active component of current.
- **For unity power factor, armature current drawn will be minimum**.
- On further increase in DC excitation, filed produced by it becomes stronger than it is required & needs demagnetisation → hence stator draws leading to demagnetise the main field → drawing leading current makes power factor leading(less than unity) → armature current increases.

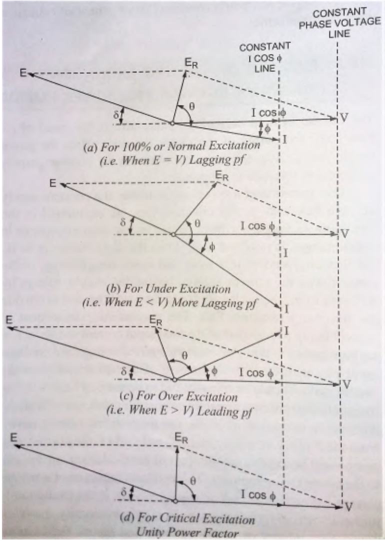


Figure 13 : Effects of varying excitation(Phasor)

(a) Normal Excitation or 100% Excitation

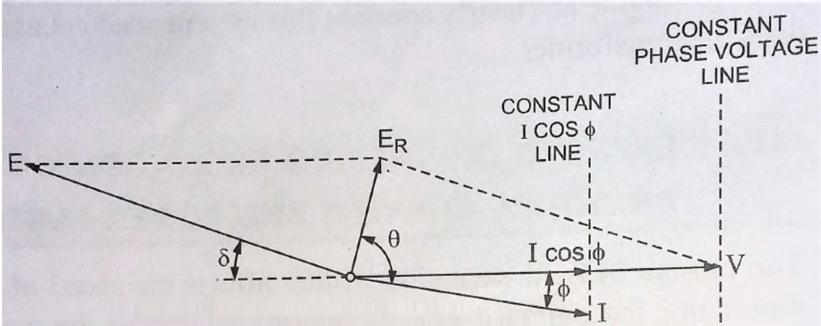


Figure 14 : Normal Excitation(Phasor)

Consider a synchronous motor loaded with constant mechanical load & operating at a constant supply voltage.

(a) **Normal Excitation or 100% Excitation**

- E = V
- Armature current(I) lags behind resultant voltage(E_R) by an angle called as **internal angle**($\theta = \tan^{-1}\left(\frac{X_s}{R_e}\right)$).
- Armature current(I) lags behind applied voltage by **power factor angle**(Φ)
- δ = **load angle** or **torque angle**
- Power drawn per phase from the supply = VIcos Φ which remains constant for constant applied voltage & constant mechanical load.
- As speed remains constant, **increase in DC excitation causes increase in excitation voltage(E) & vice-versa**

(b) Under Excitation

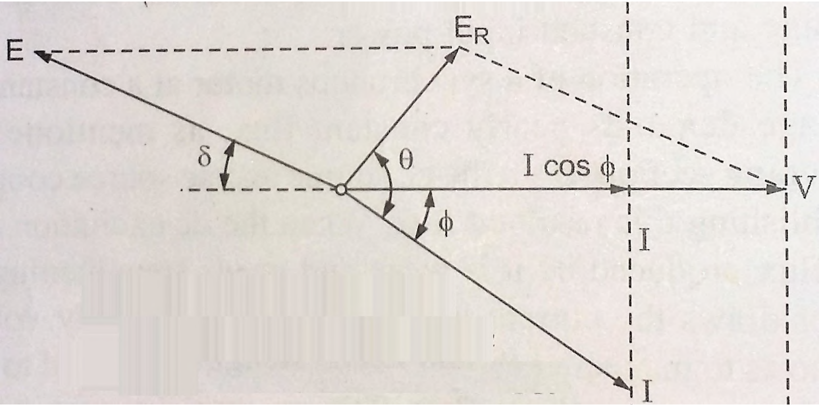


Figure 15 : Under Excitation(Phasor)

(b) **Under Excitation**

- E $<$ V
- When excitation is reduced, excitation voltage(E) decreases in magnitude $\rightarrow$ resultant voltage(E_R) is shifted in clockwise direction.
- Since internal angle(θ) is fixed, armature current phasor(I) is also shifted in clockwise direction.
- $\implies$ Power factor angle(Φ) increases $\rightarrow$ power factor($\cos\Phi$) decreases.
- Armature current(I) increases($\because$ active current component $I\cos\Phi$ is constant).
- **With reduction in DC excitation, synchronous motor draws more current from the supply at lower lagging power factor**.

(c) Over Excitation

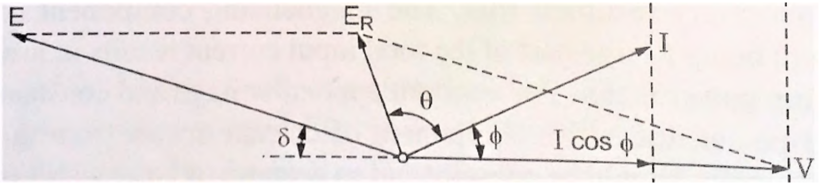


Figure 16 : Over Excitation(Phasor)

(c) **Over Excitation**

- E $>$ V
- When excitation is increased above 100%, excitation voltage(E) increases in magnitude $\rightarrow$ resultant voltage(E_R) is shifted in anticlockwise direction.
- Since internal angle(θ) is fixed, armature current phasor(I) is also shifted in anticlockwise direction.
- Armature current(I) becomes leading with respect to supply voltage(V).
- **Motor operates at leading power factor**.

(d) Critical Excitation

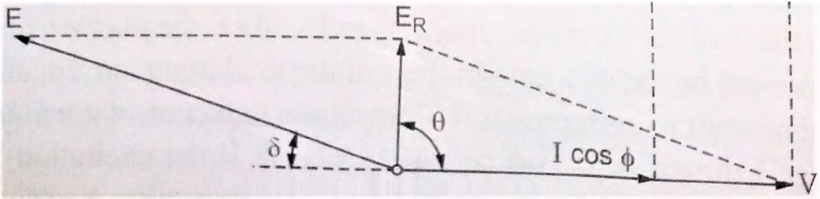


Figure 17 : Critical Excitation(Phasor)

(d) **Critical Excitation**

- For a certain value of excitation, power factor angle(Φ) between V & I becomes zero $\implies$ armature current(I) is in phase with supply voltage(V) $\rightarrow$ value of current drawn from the supply will be minimum.
- If the excitation is increased above critical excitation, motor draws current with leading power factor.

1.4 Effects of Excitation - Summary

- When the DC excitation of synchronous motor is increased above 100%, first power factor improves until it becomes unity.
- At unity power factor, current drawn from the supply is minimum.
- With further increase in DC excitation, power factor becomes leading & decreases & current drawn from the supply main increases.

Types of Excitation	**Comparison of E & V**	**Nature of power factor**	**Armature current(I)**
Normal Excitation	E = V	Lagging	Increased I
Under Excitation	E < V	Lagging	Increased I
Over Excitation	E > V	Leading	Increased I
Critical Excitation	E > V	Unity	Minimum I

Table 1 : Comparison of Various Excitations

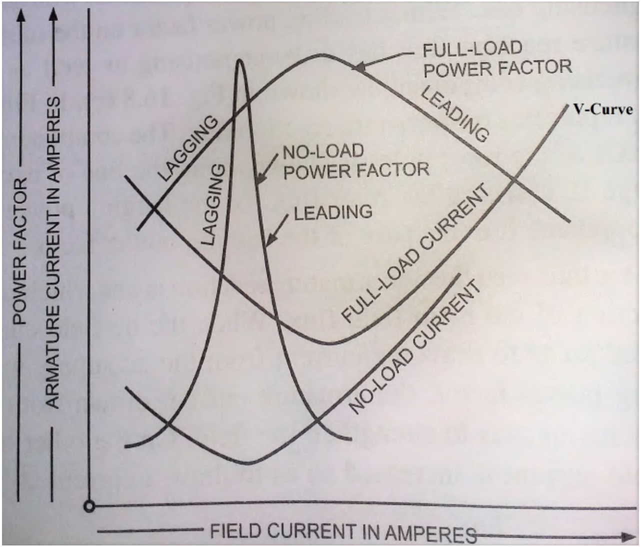


Figure 18 : V curve & Inverted V curve

1.5 V-curve

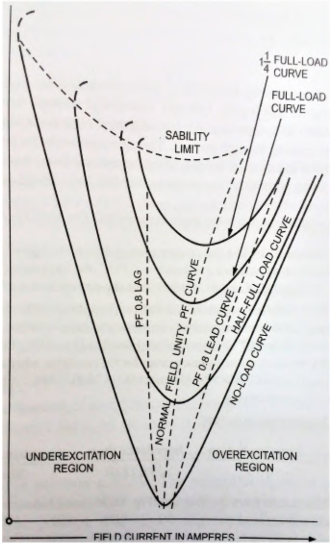


Figure 19 : V-curve of synchronous motor

- **V-curve** of a synchronous motor is the curve showing the variation of armature current with field current.
- The curve has *'V'* shape.
- V-curves of a synchronous motor give relation between armature current & field current for different power inputs.
- With low value of field current, armature current is large & lagging
- As field current is increased, power factor increases & armature current decreases until it reaches a minimum value.
- When armature current is minimum, power factor is unity.
- The lines drawn though points of equal power factor at different loads are known as **compounding curves**.
- The region in which field current is less than it's normal value is known as **region of under excitation** or **region of lag**.

1.5 V-curve

- If the field current is further increased, power factor becomes leading & begins to decrease. So armature current begins to increase.
- The region in which field current is more than the normal value is known as **region of over excitation** or **region of lead**.
- From V-curve, it is clear that no-load armature current rises sharply, with change in field current, on either side of it's unity power factor point.
- At stability limit, motor pulls out of synchronism.

1.6 Inverted V-curve

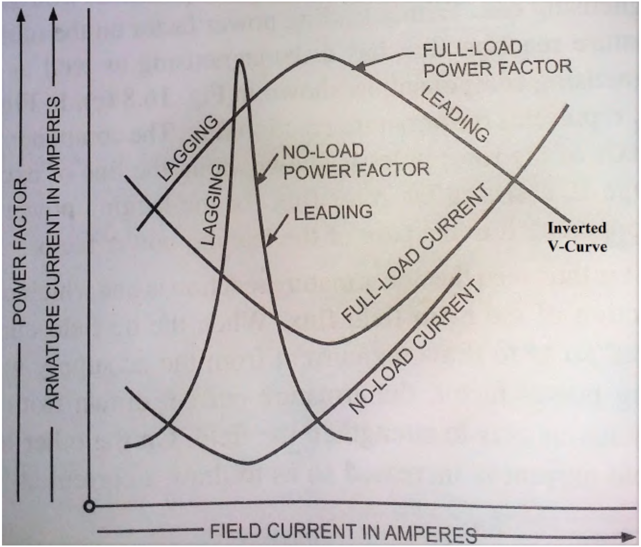


Figure 20 : Inverted V-curve of synchronous motor

1.6 Inverted V-curve

- **Inverted V-curve** of a synchronous motor is the curve showing the variation of power factor with field current.
- The curve has inverted-V shape(ie, $\wedge$).
- Inverted V-curves of a synchronous motor give relation between power factor & field current for different power inputs.
- The region in which field current is less than it's normal value is known as **region of under excitation** or **region of lag**.
- The region in which field current is more than the normal value is known as **region of over excitation** or **region of lead**.

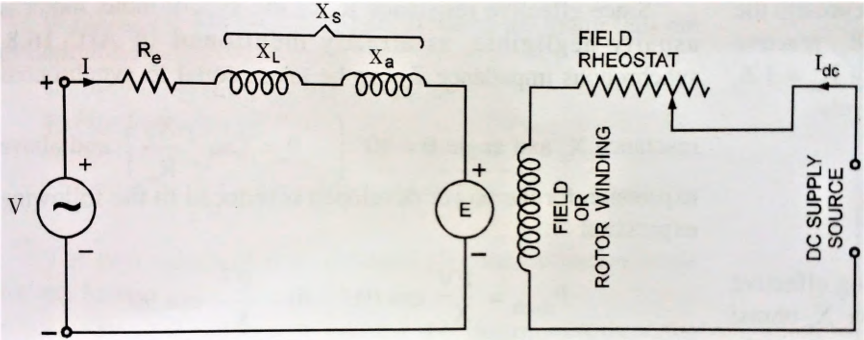


Figure 21 : Equivalent Circuit of Synchronous Motor

1.7 Equivalent Circuit

- Synchronous motor is fed from two sources of supply; field structure(rotor) from DC source & armature winding(stator) from 3-phase AC source.
- **Under normal operation, no emf is induced in rotor due to stator field because the rotor revolves at the same speed as the stator field**.
- Only DC current flows through rotor winding.
- In stator, two effects are to be considered.
  1. Rotor field cutting the stator conductors at synchronous speed(N_s) $\rightarrow$ voltage is induced in the stator winding due to field rotating at synchronous speed $\rightarrow$ induced emf is known as **excitation voltage** or **back emf** or **counter emf** $\rightarrow$ opposes the voltage applied(V) to the stator.
     $\rightarrow$ Magnitude of excitation voltage(E) depends upon rotor flux per pole.
  2. Effect of stator revolving field on stator winding $\rightarrow$ The effect is accounted by including synchronous reactance(X_s).
     $\rightarrow$ Effective resistance(R_e) is included to account for copper loss in the stator winding & stray power loss.
     $\rightarrow$ Synchronous impedance(Z_s) $= R_e + jX_s$

From equivalent circuit, Excitation voltage per phase(reversed),

$$-E = V - IZ_s$$

Armature current per phase,

$$I = \frac{V + E}{Z_s} = \frac{E_R}{Z_s}$$

Synchronous impedance,

$$Z_s = R_e + jX_s$$
$$|Z_s| = \sqrt{R_e^2 + X_s^2}$$

1.8 Phasor Diagram

- In synchronous motor, 'V' is the source voltage applied to the stator winding & E is the counter emf which is internally generated.
- Supply voltage 'V' has to overcome synchronous impedance drop(IZ_s) & counter emf(E).
- Voltage equation of synchronous motor

$$V = -E + IZ_s$$

In phasor diagram,

OA = Applied voltage(V)
OB = Load current(I)
OC = Resistive drop(IR_e)
CD = Reactive drop(IX_s)
OD = Resultant voltage($E_R = IZ_s$)
AD = Excitation voltage(E)

All are phase values

1.8 Phasor Diagram

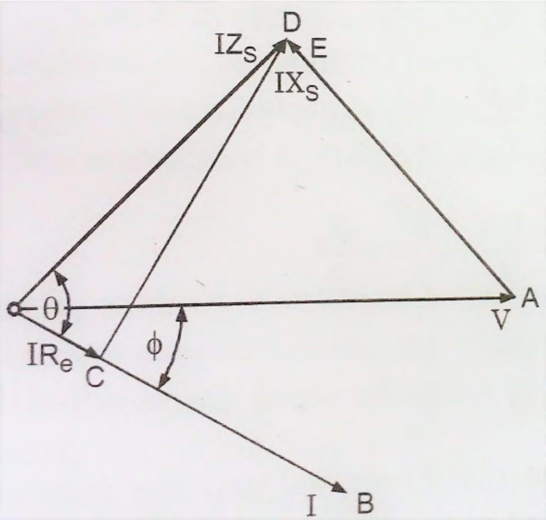


Figure 22 : Phasor diagram when Φ is -ve

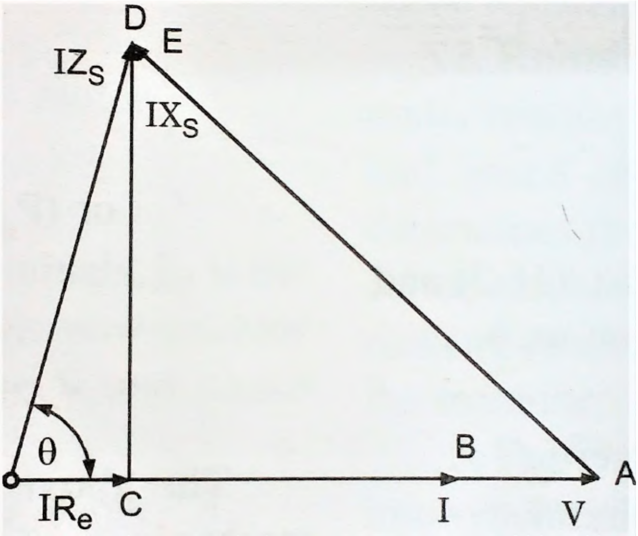


Figure 23 : Phasor diagram when $\Phi = 0$

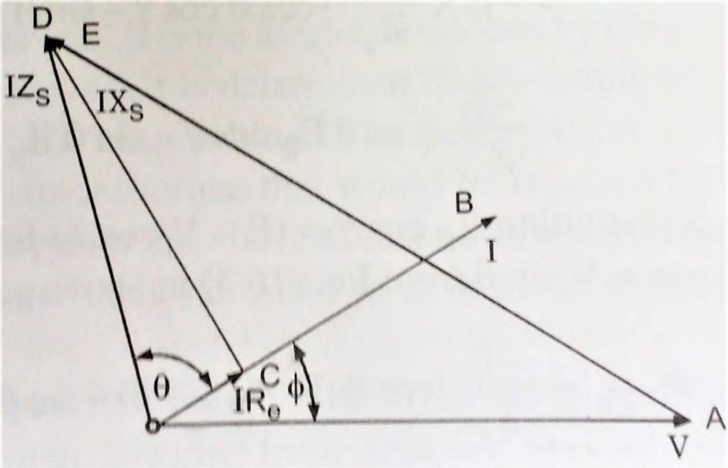


Figure 24 : Phasor diagram when Φ is +ve

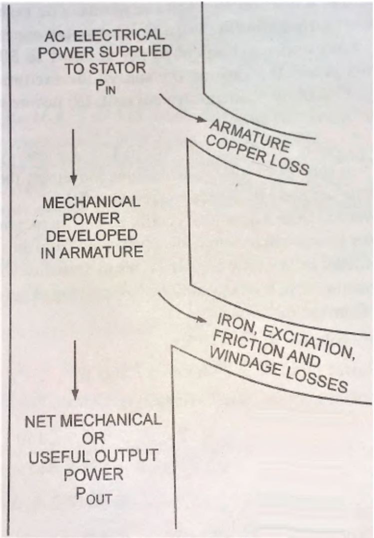


Figure 25 : Power flow diagram of synchronous motor

- Synchronous motor: electrical power from source $\rightarrow$ mechanical power output.
- Electrical power input $\rightarrow$ (armature copper loss) $\rightarrow$ mechanical power $\rightarrow$ (friction, windage, hysteresis & eddy current losses) $\rightarrow$ useful mechanical power output.
- Power supplied to stator of synchronous motor,

$$P_{in} = \sqrt{3} V_L I_L \cos \Phi$$

  V_L = Stator line voltage, I_L = stator line current
- Armature copper loss $= 3I^2 R_e$
  I = stator phase current.
- Mechanical power developed $= \sqrt{3} V_L I_L \cos \Phi$ - $3I^2 R_e$
- Net mechanical output(P_{out}) = Mechanical power developed - (friction, windage, hysteresis & eddy current losses)
- Efficiency(η) $= \frac{P_{out}}{P_{in}} \times 100$ %

Hunting & Damping

- When synchronous motor is loaded, the rotor poles falls back by a certain angle(ie, load angle δ) behind the poles of the forward rotating magnetic field of stator.
- If the load is suddenly thrown off, the rotor poles are pulled into almost exact position to the poles of the forward field, but due to moment of inertia, rotor poles travel too far. They are then pulled back again & son on.
- Thus an oscillation is set up about the equilibrium position corresponding to the new load.
- This oscillation of the rotor about it's equilibrium position is called **hunting**.
- Hunting
  - increases the probability of loosing synchronism
  - produces severe mechanical stresses & large variation in current & power drawn by the motor
  - increases losses
  - increases temperature

Causes of Hunting :

- Sudden change in load or excitation current
- Occurrence of fault in the system that is supplied by the alternator
- Cyclic variations in load torque.

Hunting can be suppressed by

- providing **damper winding** :
  - short-circuited copper bars embedded in the faces of the field poles of synchronous motor
  - When machine runs exactly at synchronous speed, no emf is induced in the damper winding $\rightarrow$ no current is flows through damper winding
  - When speed of the machine slightly deviates from N_s or hunting takes place $\rightarrow$ there is relative motion between damper winding(placed in rotor pole faces) & the rotating air gap flux $\rightarrow$ emf & currents are set up in damper winding $\rightarrow$ opposes $\rightarrow$ helps to suppress the oscillations
- using **flywheels**
- designing synchronous machines with suitable synchronising power coefficients.

Q1. A 3-phase, 440V, 50Hz star connected synchronous motor takes 7.46kW from the 3-phase supply mains. The resistance per phase of the armature winding is 0.5Ω. The motor operates at a pf of 0.75 lag. Iron & mechanical losses amount to 500W. The excitation loss is 650W. Assume the source for excitation to be a separate one. Calculate (i) armature current (ii) power supplied to motor (iii) efficiency.

Ans:

(i) Armature current, I = 13.05A
(ii) Total power supplied to motor = 8110W
(iii) Efficiency = 82.67%

Q2. A 3.3kV delta connected motor has synchronous reactance per phase of 18Ω. It operates at a leading pf of 0.707 when drawing 800kW from mains. Calculate it's excitation emf.

Ans:
(i) Excitation emf, $E = 4972.8V$

Introduction

- Most widely used AC motor
    - Low cost
    - Simple and rugged construction
    - Absence of commutator
    - Good power factor
    - High efficiency
    - Good speed regulation
- **Singly excited machine**
    - Stator winding is connected to AC supply
    - No electrical connection from rotor to any source of supply
- AC supply $\rightarrow$ stator currents $\rightarrow$ stator field $\rightarrow$ rotor emf and current $\rightarrow$ net unidirectional torque
- **Asynchronous motors**
  $\implies$ speed of motor(N) $\neq$ synchronous speed(N_s)

- Machine is **reversible** $\rightarrow$ machine can be operated as a motor or a generator
- Power is transferred from stator to rotor by mutual induction $\rightarrow$ hence the name **Induction Machine**

- Laminated **stator** core carrying polyphase winding
- Laminated **rotor** core carrying either a squirrel cage or polyphase winding (Non-salient pole construction is used for all polyphase induction motors)

1. Frame

- Outer body
- Supports stator core & winding
- Protects inner parts of the machine
- Ventilating housing
- Frame may be die-cast(greater mechanical strength) or fabricated
- Frame is provided with feet by which machine is fixed to the base plate

2.1 Construction

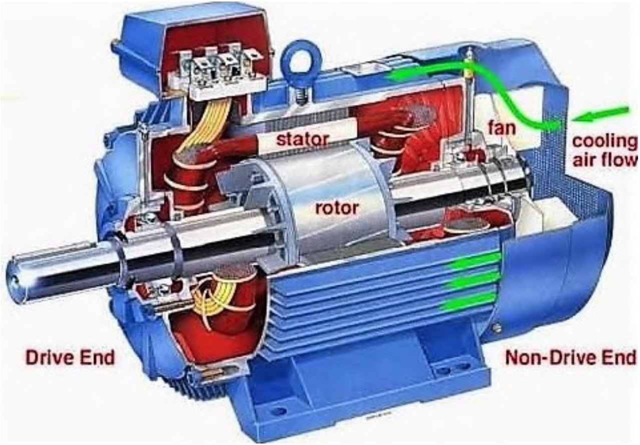


Figure 26 : Induction motor

2.1 Construction

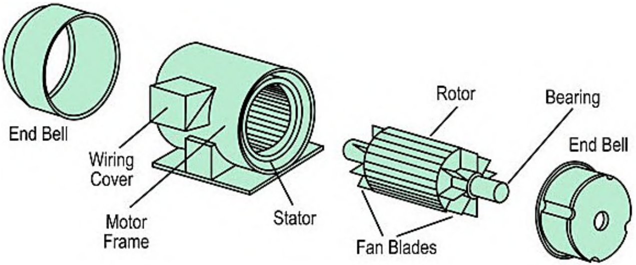


Figure 27 : Induction motor

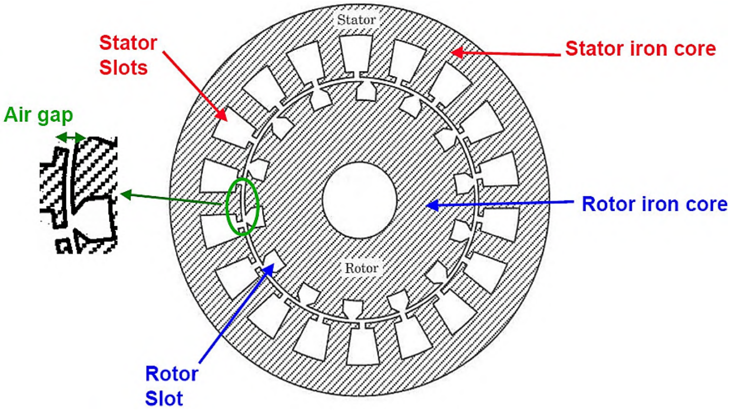


Figure 28 : Induction motor

2. Stator core

- Carry alternating flux
- Assembled of high grade silicon steel punchings of(0.35mm to 0.65mm)
- Punchings are insulated with **varnish**
- Radial **ventilating ducts** are provided
- **Slots(open or semi-closed)** are punched on the inner periphery to accommodate stator winding

- Air-gap is made as small as practicable(0.3mm to 0.35mm $\rightarrow$ small machine & 1mm to 1.5mm $\rightarrow$ high power machine)

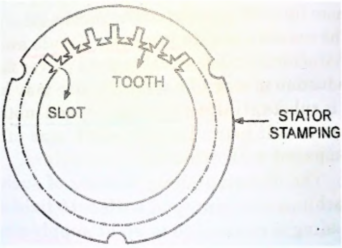


Figure 29 : Stator core

3. Stator or Field Winding

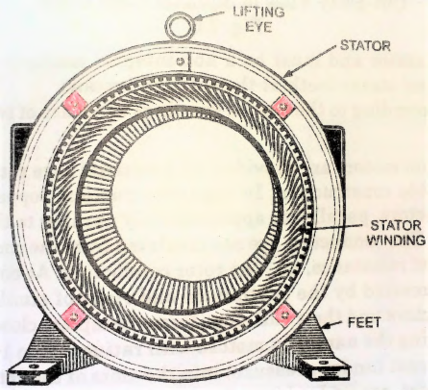


Figure 30 : Stator

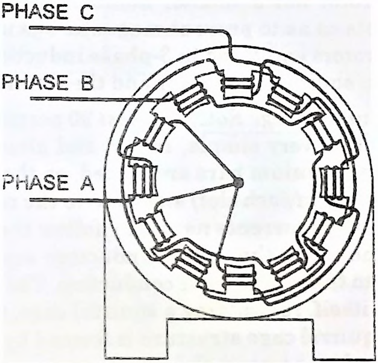


Figure 31 : 3-phase 4-pole star connected stator winding

2.1 Construction

- Three phase winding either **star or delta connected**
- Winding is wound for a definite number of poles as per requirement of speed
- Usually double layer short-pitched winding
- Winding is placed in the stator slots

4. Rotor

- Cylindrical laminated iron core
- Slots are punched on the outer periphery of rotor core
- Rotor conductors are placed in rotor slots
- Laminated silicon sheets are employed for rotor core
- Ventilating ducts are also provided
- No. of rotor slots < no. of stator slots(rotor slots = non-integral multiple of stator slots)

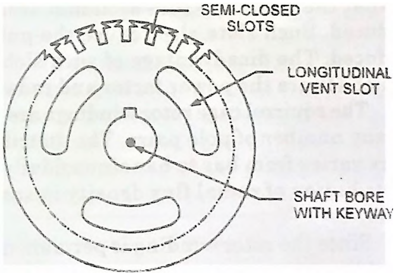


Figure 32 : Rotor stamping

- Two types of rotors

a. Squirrel Cage Rotor

- Simple & robust construction
- **Copper bars** are placed as rotor conductors, approximately parallel to shaft & close to rotor surface
- Ends of all rotor conductors are short-circuited using **end rings**
- Rotor slots are usually semi-closed or closed type
- Since rotor conductors are **permanently** short-circuited using end rings, addition of external resistance is not possible.

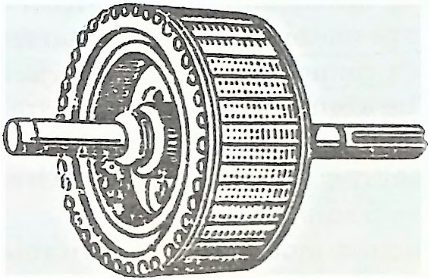


Figure 33 : Squirrel cage rotor

b. Wound Rotor

- Polyphase winding is placed in rotor slots
- Smaller number of rotor slots & fewer turns per phase
- Rotor is wound for the same number of poles as that of stator
- Rotor winding may be star or delta connected(usually star)
- Finish terminal are connected together → star-point
- Start terminal are connected to slip rings → brushes

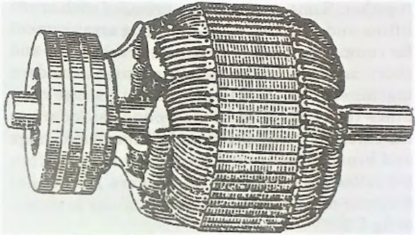


Figure 34 : Wound rotor

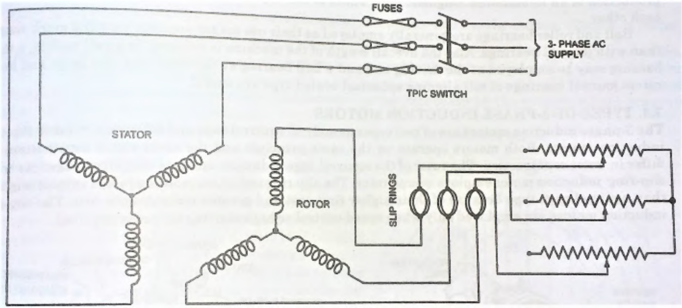


Figure 35 : Slip ring induction motor with starting rheostat

- During starting → maximum resistance is included in the rotor circuit
- Resistance is gradually cut out as the motor picks up speed
- During normal ruining condition → entire external resistance is cut out & the rotor windings are short-circuited
- Wound rotor induction motors are called as **Slip-Ring Induction Motors**

5. Shaft & Bearings

- Rotor shaft is supported by bearings
- Ball and roller bearings are usually used

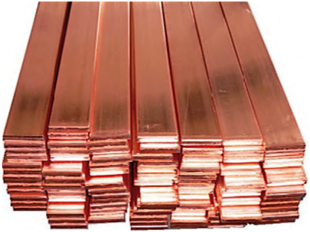


Figure 36 : Copper bars

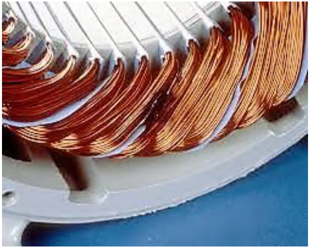


Figure 37 : Winding

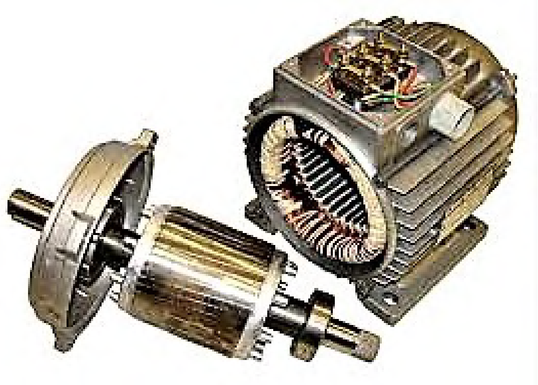


Figure 38 : Squirrel cage induction motor

2.1 Construction

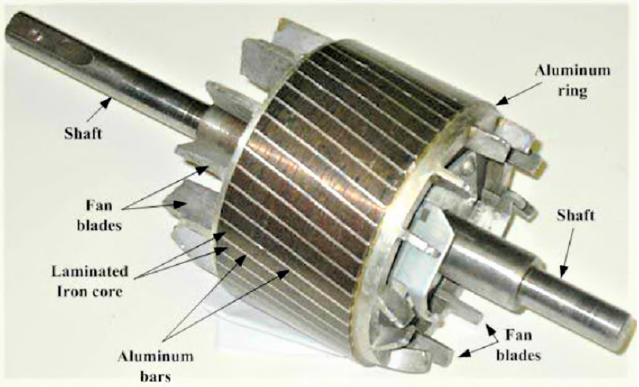


Figure 39 : Squirrel cage rotor

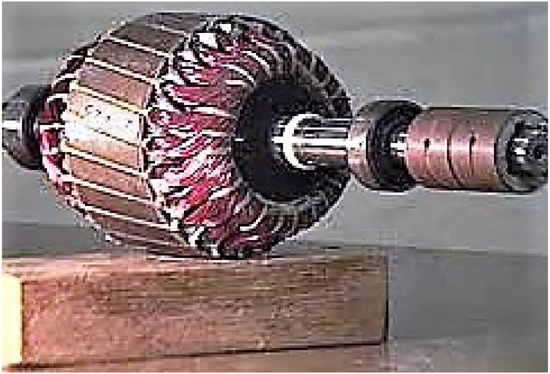


Figure 40 : Wound rotor

Figure 41 : Wound rotor

2.2 Types

1. **Squirrel Cage Induction Motor**
2. **Wound Rotor or Slip-ring Induction Motor**

- Same principle of operation & stator construction
- Differ in rotor construction
- Squirrel cage induction motor $\rightarrow$ Low cost
- Slip-ring Induction Motors are used when speed control or high starting torque is required

- Stator winding is connected to a 3-phase AC supply.
- **Rotating magnetic field** is established.
- Rotating magnetic field rotates at **synchronous speed(N_s)**.
- Direction of revolution of field depends on the phase sequence of the supply.
- Direction of rotation of field can be reversed by interchanging any two terminals of the supply.
- Synchronous speed(N_s) $= \frac{120f}{P}$
- f = supply frequency, P = number of poles on stator.
- As the rotating magnetic field sweeps across the rotor conductors, an emf is induced in these conductors due to **electromagnetic induction**.
- Since the rotor winding is either directly shorted or closed through external resistance, current is circulated through rotor winding.

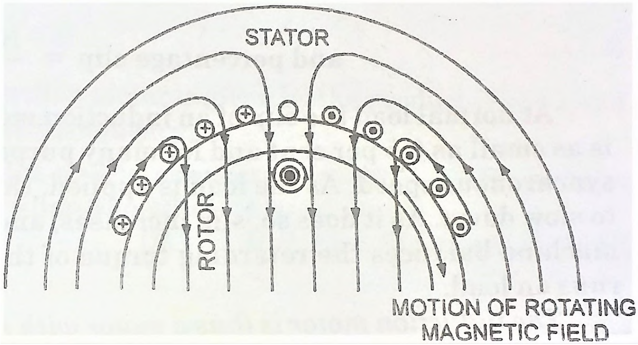


Figure 42 : Stator field and rotor

- Production of torque
  - Let's assume that field rotates in clockwise direction and rotor is stationary.
  - Current carrying conductor placed in magnetic filed experiences force.

2.3 Principle of Operation

- Rotor conductor experiences a force tending to rotate the conductor towards right(by applying left hand rule).
- After one half-cycle, both stator current and rotor current will have reversed $\rightarrow$ hence rotor tends to rotate in the same direction.
- If the developed torque overcomes resisting torque, rotor will accelerate in the same direction as the rotation of stator field.
- **Lenz's Law** $\rightarrow$ Direction of induced emf would be in such a direction that it would try to oppose the very cause for which it is due.
- Relative speed between the rotating field and stationary rotor conductors causes the production of induced emf and current in rotor.
- Hence the torque produced due to induced rotor currents will cause the rotor to follow the rotating magnetic field and thus reducing the relative speed.

- When the machine is about to start, relative speed between stator field & stationary rotor is synchronous speed(N_s) $\implies$ frequency of rotor induced emf(f_r) = supply frequency(f)
- As the motor picks up speed, relative speed decreases.
- The magnitude of rotor induced emf, induced rotor current and the torque depends on the relative speed.
- Hence when relative speed is zero(ie, rotor runs at synchronous speed), there would be no induced emf & no induced rotor currents, no rotor field and hence no torque. Hence an **induction motor can not run at synchronous speed**

- At no-load, induction motor runs at a speed very close to N_s $\rightarrow$ small relative speed $\rightarrow$ small rotor induced emf and current $\rightarrow$ small torque just sufficient to overcome the losses and maintain the rotor in motion.
- As load is applied $\rightarrow$ rotor will slow down as the torque developed at no-load will not be sufficient to keep the rotor rotating at no-load speed against additional opposing torque of load.
- Relative speed between stator field and rotor is increased $\rightarrow$ greater rotor induced emf and current $\rightarrow$ greater torque.
- When the induction motor is loaded, the motor slows down until the relative motion between rotor & rotating field is just sufficient to result in the development of torque necessary for that particular load.

Slip

- Speed(N) of polyphase induction motor is always less than synchronous speed(N_s).
- **Slip** is defined as the difference between speed of stator field and actual speed of rotor usually expressed as percentage of synchronous speed.

$$\% \, Slip = \frac{(N_s - N)}{N_s} \times 100$$

- Normal slip is usually between 2% and 5%.
- No-load slip is usually as small as 0.5%.
- As load is applied, slip increases from no-load slip value.

Frequency of Rotor Current or EMF(f_r)

- The rate at which rotor conductors are being cut by the rotating flux depends upon the relative speed between rotating magnetic field & rotor(ie, **slip speed**).
- Frequency of rotor current(or emf) induced by relative motion between rotor conductors and the stator revolving magnetic field,

$$f_r = \frac{Relative\ speed\ in\ rpm}{120/P}$$

$$= \frac{N_s - N}{120/P}$$

$$But\ N_s - N = s \times N_s = s \times \frac{120f}{P}$$

$$\implies f_r = s \times \frac{120f}{P} \times \frac{P}{120} = sf$$

- $\implies$ **Slip frequency,** $f_r = \mathbf{sf}$

- At standstill condition, 3-phase induction motor is equivalent to 3-phase transformer with secondary short-circuited.
- $\therefore$ rotor induced emf per phase(E_2), at standstill condition,

$$E_2 = E_1 \times \frac{N_2}{N_1}$$

where E_1 = Applied voltage per phase to stator, N_2 = Number of turns per phase on rotor, N_1 = Number of turns per phase on stator

- When rotor rotates, the relative speed of rotor with respect to stator field drops in proportion to slip(s).
- $\therefore$ **Rotor induced emf** $= s \times E_2$
- ie, for slip s, **induced emf in rotor is 's' times the induced emf in the rotor at standstill condition.**

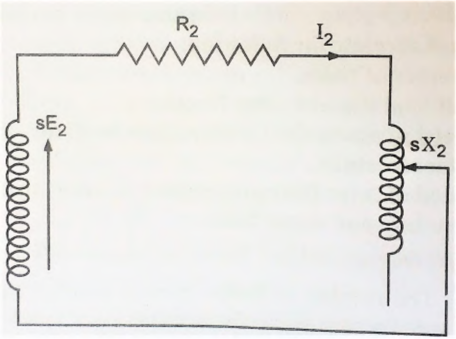


Figure 43 : Circuit diagram of induction motor rotor

Let,

R_2 = rotor resistance per phase
L_2 = rotor inductance per phase
E_2 = rotor induced emf per phase at standstill

At standstill,

$$\begin{aligned}
Induced\ emf\ \ per\ phase\ in\ rotor &= E_2 \\
Rotor\ winding\ resistance\ per\ phase &= R_2 \\
Rotor\ winding\ reactance\ per\ phase, X_2 &= 2\pi f L_2 \\
Rotor\ winding\ impedance\ per\ phase, Z_2 &= \sqrt{R_2^2 + X_2^2} \\
Rotor\ current\ per\ phase, I_2 &= \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \\
Power\ factor\ of\ rotor\ current, Cos\Phi_2 &= \frac{R_2}{\sqrt{R_2^2 + X_2^2}}
\end{aligned}$$

At slip s,

$$\begin{aligned}
Induced\ emf\ \ per\ phase\ in\ rotor &= sE_2 \\
Rotor\ winding\ resistance\ per\ phase &= R_2 \\
Rotor\ winding\ reactance\ per\ phase &= 2\pi f_r L_2 = 2\pi s f L_2 = sX_2 \\
Rotor\ winding\ impedance\ per\ phase, Z_2 &= \sqrt{R_2^2 + (sX_2)^2} \\
Rotor\ current\ per\ phase, I_2 &= \frac{sE_2}{Z_2} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \\
&= \frac{E_2}{\sqrt{\left(\frac{R_2}{s}\right)^2 + X_2^2}} \\
&= \frac{E_2}{\sqrt{\left(R_2 + R_2\frac{(1-s)}{s}\right)^2 + X_2^2}}
\end{aligned}$$

- $R_2\frac{(1-s)}{s}$ = fictitious resistance representing **electrical load** on the rotor.
- Power consumed by fictitious resistance $= I_2^2 R_2 \frac{(1-s)}{s}$ and is converted into mechanical power.

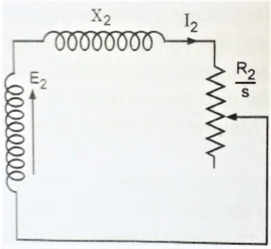


Figure 44 : Circuit of induction motor rotor

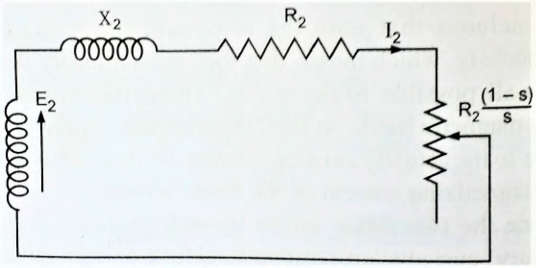


Figure 45 : Circuit of induction motor rotor

$$Power\ factor\ of\ rotor\ current,\ Cos\Phi_2 = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Rotor Torque

- Torque(T) developed by the rotor of induction motor is directly proportional to
  1. Rotor current(I_2)
  2. Stator flux per pole(Φ)
  3. Power factor of the rotor circuit($Cos\Phi_2$)

$$\implies T \propto \Phi\ I_2\ Cos\Phi_2$$

But rotor emf per phase at standstill, $E_2 \propto \Phi$

$$\therefore T \propto E_2\ I_2\ Cos\Phi_2$$

$$\implies T = K\ E_2\ I_2\ Cos\Phi_2, \qquad where\ K = any\ constant$$

$$
\begin{aligned}
Torque(T) &= K\ E_2\ I_2\ Cos\Phi_2 \\
&= K\ E_2 \left(\frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \right) \times \left(\frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}} \right) \\
&= \frac{K\ s\ R_2\ E_2^2}{R_2^2 + (sX_2)^2}
\end{aligned}
$$

- Running torque $\propto$ square of supply voltage($\because$ $E_2 \propto$ supply voltage).

- Condition for maximum torque, $s = \frac{R_2}{X_2} \implies R_2 = sX_2$
- $T_{max} = \frac{K\ E_2^2}{2\ X_2}$
- **Maximum torque(T_{max})** is also known as **breakdown torque** or **pull-out torque**
- T_{max} **is not affected by change in rotor resistance. But the value of slip at which maximum torque occurs changes with change in rotor resistance.**

Starting Torque(T_{st})

- At starting, s = 1

$$T_{st} = \frac{K\ R_2\ E_2^2}{R_2^2 + X_2^2}$$

- Usually supply voltage(V) is constant. Hence the stator flux(Φ) and E_2 are also constant.

$$\therefore\ T_{st} = \frac{K_1\ R_2}{R_2^2 + X_2^2} = \frac{K_1\ R_2}{Z_2^2}$$

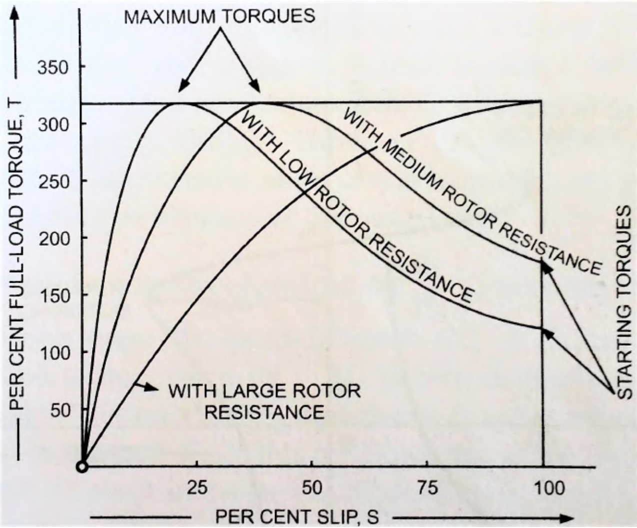


Figure 46 : Torque-Slip Characteristics

- Torque developed by the induction motor,

$$T = \frac{K\ s\ R_2\ E_2^2}{R_2^2 + (sX_2)^2}$$

1. When s = 0, T = 0 $\implies$ Torque-slip curve **starts from origin**.
2. When speed is close to N_s, slip(s) will be very low.
   $\implies$ sX_2 can be neglected in comparison with R_2.
   $\implies$ T $\propto$ slip, if R_2 is constant.
   $\implies$ At speeds near N_s, Torque-slip curve is **approximately straight line**.
3. With increase in applied load, speed(N) decreases $\rightarrow$ slip(s) increases $\rightarrow$ torque increases and reaches maximum torque(T_{st}) when s=$\frac{R_2}{X_2}$.
   - Slip corresponding to maximum torque $\rightarrow$ **breakdown slip(s_b)**
4. With further increase in slip due to increase in load beyond maximum torque, $\rightarrow$ torque decreases $\rightarrow$ motor slows down $\rightarrow$ eventually motor stops.

5. Motor operates for the value of slip, $\mathbf{0} < \mathbf{s} < s_b$
6. With higher values of slip(s), $R_2 << sX_2$. Hence R_2 may be neglected.
   $\implies$ T $\propto \frac{s}{(sX_2)^2} \propto \frac{1}{s}$ if standstill reactance(X_2) is constant.
   $\implies$ Torque-slip curve is **rectangular hyperbola**
   for the region s $> s_b$

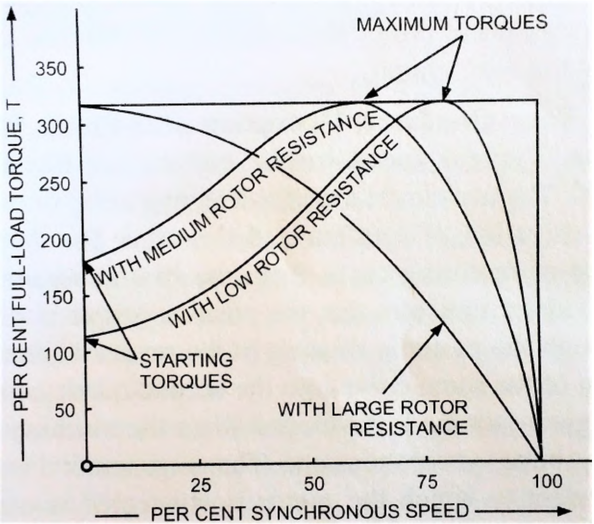


Figure 47 : Torque-Speed Characteristics

Torque-Speed Curve & Operating Region

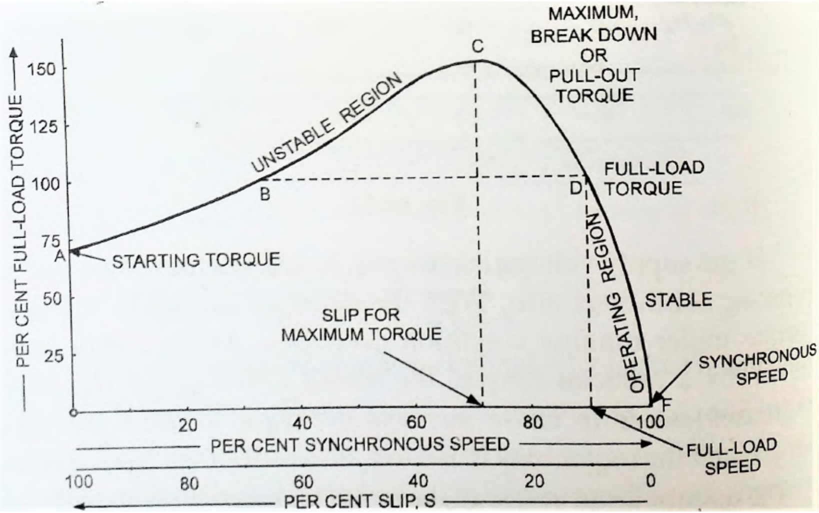


Figure 48 : Torque-Speed Curve

Torque-Speed Curve & Operating Region

- For any load torque, there are two operating points 'B' and 'D'.
- At point 'B' $\rightarrow$ operation is **unstable**($\because$ any tendency of speed rise increases developed torque beyond load torque causing further rise in speed).
- At point 'D' $\rightarrow$ operation is **stable**. Because,
    - Tendency to rise in speed will be opposed by decrease in developed torque.
    - Tendency to fall in speed causes increase in developed torque to bring the motor to operating point 'D'.
- Region AC $\rightarrow$ **unstable region of operation**
- Region CF $\rightarrow$ **stable region of operation**

Torque-Speed Curve & Operating Region

- At full load, motor runs at a speed N rpm
- If the load is increased $\rightarrow$ motor speed drops till the torque developed by the motor equals load torque.
- As long as load torque $=$ developed torque, motor will run at constant speed.
- If load torque $>$ breakdown torque, motor immediately stops.
- T_{max} ranges from 200 to 300% of $T_{full-load}$.

Torque and Power

Let,

T_{sh} = shaft torque, T_g = gross torque, N = rotational speed

- Power output

$$P_{out} = T_{sh} \times \frac{2\pi N}{60} \quad watts$$

- Mechanical power developed

$$P_{mech} = T_g \times \frac{2\pi N}{60} \quad watts$$

$$\implies \quad T_{sh} = 9.55 \times \frac{P_{out}}{N} \quad Nm$$

$$T_g = 9.55 \times \frac{P_{mech}}{N} \quad Nm$$

- (T_g - T_{sh}) $\rightarrow$ torque lost due to friction & windage losses in motor.

2.5 Equivalent Circuit of Induction Motor

- At standstill, induction motor can be considered as a transformer with it's 2^o winding short-circuited.
- In transformer, load on 2^o is electrical.
- In induction motor, load on the rotor is mechanical which can be represented by the equivalent electrical load R'_L

$$R'_L = \frac{R_2}{K^2}\left(\frac{1-s}{s}\right) = R'_2\left(\frac{1-s}{s}\right)$$

where R_2 = rotor resistance per phase, K = turns ratio

2.5 Equivalent Circuit of Induction Motor

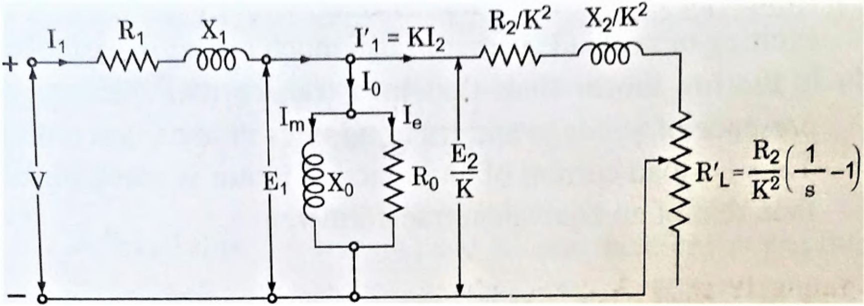


Figure 49 : Equivalent circuit of induction motor

V = Applied voltage per phase
R_1 = Stator resistance per phase
X_1 = Stator leakage reactance per phase
R_2 = Rotor resistance per phase
X_2 = Rotor standstill leakage reactance per phase
R_o = No-load resistance per phase
X_o = No-load reactance per phase
E_1 = emf induced per phase in stator
E_2 = emf induced per phase in the rotor

- The circuit can be simplified by transferring no-load current components to the supply terminals.
- At no-load, stator impedance drop($I_o Z_1$), being very small can be neglected.

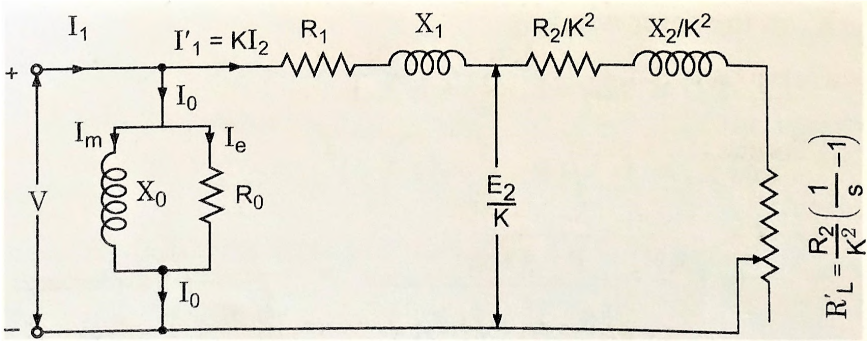


Figure 50 : Equivalent circuit of induction motor

- Energy component of no-load current(I_o)

$$I_e = \frac{V}{R_o}$$

- magnetising component of no-load current(I_o)

$$I_m = \frac{V}{X_o}$$

2.6 Phasor Diagram

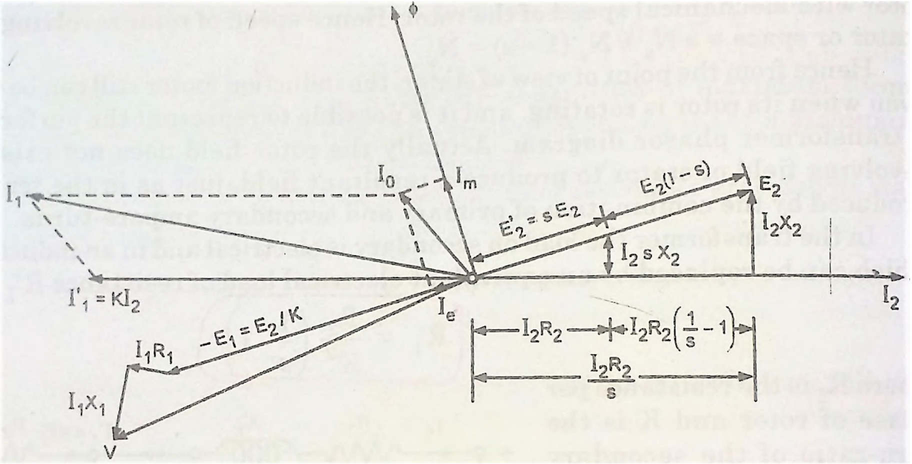
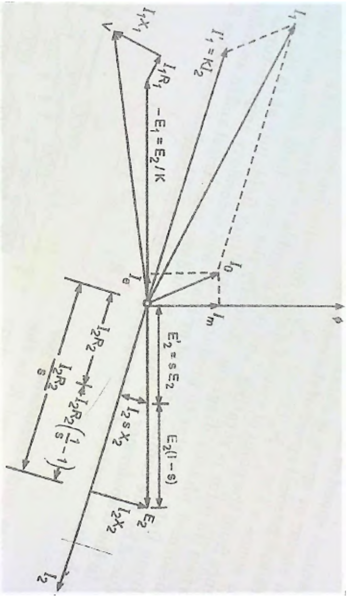


Figure 51 : Phasor diagram of induction motor


V
I_1X_1
I_1R_1
$-E_1 = E_2/K$
$I_1'=KI_2$
I_1
I_0
I_e
I_m
ϕ
$E_2' = sE_2$
$E_2(1-s)$
E_2
I_2sX_2
I_2X_2
I_2R_2
$\frac{I_2R_2}{s}$
$I_2R_2\left(\frac{1}{s}-1\right)$
I_2

- At any load, when motor is operated with slip s, induced emf in rotor winding,

$$\overrightarrow{E_2'} = s\overrightarrow{E_2} = \overrightarrow{I_2}(R_2 + jsX_2)$$
$$\implies sE_2 = I_2\sqrt{R_2^2 + (sX_2)^2}$$
$$\implies I_2 = \frac{E_2}{\sqrt{\left(\frac{R_2}{s}\right)^2 + X_2^2}}$$

- Total power supplied to rotor $= 3\ I_2^2 \frac{R_2}{s}$
- Power lost in rotor $= 3\ I_2^2 R_2$
- Mechanical power developed $= 3\ I_2^2 R_2\ \left(\frac{1-s}{s}\right) = 3\ I_2^2 R_L$

- $(E_2 - sE_2) = E_2$(1-s) is known as **speed emf**
- $E_1 = \frac{E_2}{K}$
- The air gap flux Φ leads induced emfs E_1 & E_2 by 90^o
- Stator current $\overrightarrow{I_1} = \overrightarrow{I_1'} + \overrightarrow{I_o}$
- $\overrightarrow{I_1'} = K\overrightarrow{I_2}$
- I_1R_1 = resistance drop per phase in stator
- I_1X_1 = reactance drop per phase in stator
- $\overrightarrow{V} = -\overrightarrow{E_1} + \overrightarrow{I_1}R_1 + \overrightarrow{I_1}X_1$
- $\overrightarrow{E_2} = \overrightarrow{I_2}R_2 + \overrightarrow{I_2}R_2\left(\frac{1-s}{s}\right) + \overrightarrow{I_2}X_2$

Q1. Find the running speed of a 4-pole induction motor working on 50Hz supply having 3% slip.

Ans:

N = 1455rpm

Q2. A 6-pole induction motor is supplied by a 10-pole alternator which is driven at 600rpm. If the motor is running at 970rpm, determine the percentage slip.

Ans:
$\%$ slip $= 3\%$

Q3. A 3-phase induction motor has 2 poles and is connected to 400V, 50Hz supply. Calculate the actual rotor speed & rotor frequency when the slip is 4%.

Ans:
$N = 2880\text{rpm}$
$f_r = 2\text{Hz}$

Q4. A 1.1kV, 50Hz delta connected induction motor has a star connected slip-ring rotor with a phase transformation ratio of (1/3.8). The rotor resistance and standstill leakage reactance are 0.012Ω & 0.25Ω per phase respectively. Neglecting stator impedance & magnetising current, determine (i) Rotor current and rotor power factor at start with slip-rings shorted. (ii) Rotor current & rotor power factor at 4% slip with slip rings shorted. (iii) External rotor resistance per phase required to obtain a starting current of 100A in the stator supply lines.

Ans:

(i) $I_2 = 1156.6A,\ Cos\Phi_2 = 0.048$ lag

(ii) $I_2 = 742.3A,\ Cos\Phi_2 = 0.768$ lag

(iii) External resistance required per phase $= 1.2836\Omega$

Q5. A 440V, 50Hz, 6 pole, three phase induction motor draws an input power of 76kW from the mains. The rotor emf makes 120 complete cycles per minute. It's stator losses are 1kW & rotor current per phase is 62A. Calculate (i) Rotor copper losses per phase (ii) Rotor resistance per phase (iii) Torque developed.

Ans:
(i) $I_2 = 62A$
(ii) $R_2 = 0.26\Omega$
(iii) $T = 716.2Nm$

Q6. A 3-phase 6-pole, 50Hz induction motor develops 3.7kW at 950rpm. What is the stator input if the stator loss is 300W?.

Ans:

Stator input = 4.195kW

Q7. A 400V, 3-phase delta connected induction motor gave the following test data

No load test	:	400V	3A	645W
Blocked rotor test	:	200V	12A	1660W

The friction & windage losses amount to 183W. Obtain the equivalent circuit diagram of the induction motor. Take stator winding resistance per phase = 5Ω.

Ans:

$I_e = 0.6A$

$I_m = 2.94A$

$R_o = 1155\Omega$

$X_o = 236\Omega$

$Z_{01} = 28.87\Omega$ per phase

$R_{01} = 11.53\Omega$ per phase

$X_{01} = 26.5\Omega$ per phase

Q8. A 110V, 3-phase star connected induction motor takes 25A at a line voltage of 30V with rotor locked. With this line voltage, power input to the motor is 440W & core loss is 40W. The DC resistance between a pair of stator terminals is 0.1Ω. If the ratio of AC to DC resistance is 1.6, find the equivalent leakage reactance per phase of the motor, stator & rotor resistance per phase.

Ans:

$R_1 = 0.08\Omega$

$R_{01} = 0.213\Omega$

$R_2' = 0.133\Omega$

$X_01 = 0.66\Omega$

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Thank You

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Synchronous and Induction Machines

Synchronous and Induction Machines (EE-202)

by

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Module 5 - Overview

1. Induction motor
   - Tests on Induction Motor
   - Circle diagram
   - Cogging, Crawling & Noise production
2. Double Cage Induction Motor
   - Torque-Speed Curve
   - Equivalent Circuit
3. Starting of Induction Motor
   - DOL Starter, Autotransformer Starter, Star-Delta Starter & Rotor Resistance Starter
4. Braking of Induction Motors
   - Plugging, Dynamic Braking & Regenerative Braking
5. Speed Control of Induction Motor
   - Stator Voltage Control, V/f Control & Rotor Resistance Control

Tests

1. **Stator Resistance Test**
2. **No-load Test**
    - Stator core loss, friction & windage losses can be determined.
3. **Blocked Rotor Test**
    - Stator & rotor copper losses can be determined.
4. **Voltage Ratio Test**
5. **Heat Run Test or Temperature Rise Test**
    - Temperature rise of different parts of motor while running at rated speed can be determined.

Stator Resistance Test

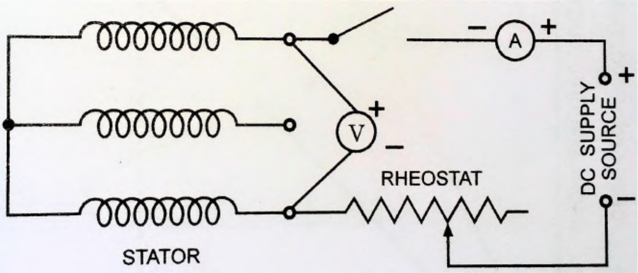

Figure 1 : Circuit diagram

- R_1 can be measured by Voltmeter-Ammeter Method.
- DC resistance between a pair of terminals is measured.
- Resistance of each phase, $R_{1dc} = \left(\frac{V/I}{2}\right)$ ohms
- Effective AC Resistance per phase$(R_1) = (1.25\ to\ 1.75)\ times\ R_{1dc}$

- No-load test is conducted to determine
  - No-load current(I_0)
  - No-load power factor($Cos\Phi_o$)
  - Windage & frictional losses(P_{wf})
  - Core loss(P_i)
  - No-load power input(P_0)
  - No-load resistance(R_0)
  - No-load reactance(X_0)
- No-load test is performed with different values of applied voltage below & above rated voltage.
- Wattmeter $\rightarrow$ Total power input(P_o)
- Voltmeter $\rightarrow$ Applied line voltage(V).
- Ammeter $\rightarrow$ No-load input current(I_o).
- **At no-load, power input(P_o) = core loss(P_i) + stator copper loss at no-load(P_{c1}) + windage & friction losses(P_{wf}).**
- $I_o \approx 30\%$ to 40% of full load current.

- Mechanical power developed during no-load test corresponds to windage & friction losses.
- At no-load, slip is extremely small(in the range of 0.001). Hence f_r is very small.
- $R_2'\left(\frac{1-s}{s}\right)$ becomes very high $\implies$ rotor circuit is **practically open at no-load.**

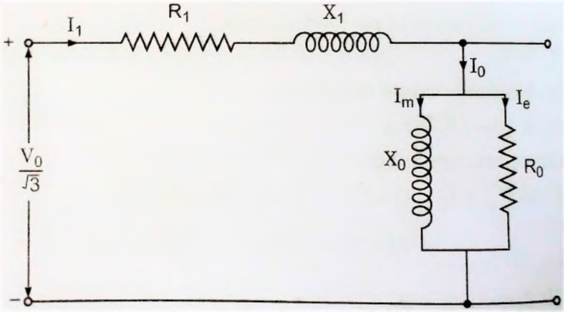

Figure 2 : Equivalent circuit of induction motor at no-load

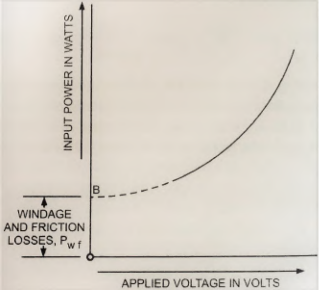

Figure 3 : P_o - V Curve

- To determine P_{wf}

- P_o - V curve is drawn & is extended to intersect vertical axis(ie, point 'B').
- V = 0 at point 'B' $\implies P_{c1} = 0$ & $P_i = 0$.
- $\therefore$ **OB = windage & friction losses(P_{wf}).**

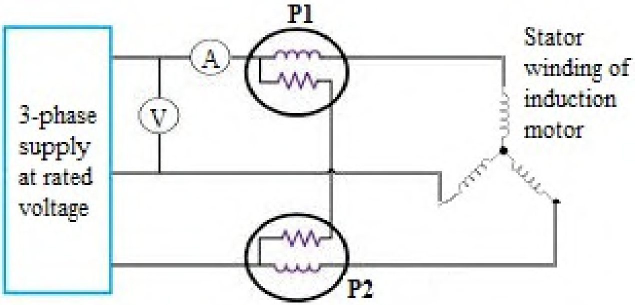

Figure 4 : No-load Test Circuit

- Voltmeter reading $\rightarrow V_o$
- Ammeter reading $\rightarrow I_o$
- No-load power input $\rightarrow P_o = P_1 + P_2$

- Let's consider the **stator is star connected.**
- Stator copper loss at no-load, $P_{c1} = 3I_o^2 R_1$
- Total core loss(P_i) = Total power input at no-load(P_o) - P_{c1} - P_{wf}

$$No-load\ power\ factor, Cos\Phi_o = \frac{P_i}{\sqrt{3}V_o I_o}$$

$$Energy\ component\ of\ no-load\ current,$$

$$I_e = I_o Cos\Phi_o = I_o \times \frac{P_i}{\sqrt{3}V_o I_o} = \frac{P_i}{\sqrt{3}V_o}$$

$$Magnetising\ component\ of\ no-load\ current, I_m = \sqrt{I_o^2 - I_e^2}$$

$$No-load\ impedance, Z_o = \frac{V_o/\sqrt{3}}{I_o}$$

$$No-load\ resistance, R_o = \frac{V_o/\sqrt{3}}{I_e}$$

$$No-load\ reactnce, X_o = \sqrt{Z_o^2 - R_o^2}$$

Blocked Rotor Test

- Blocked Rotor Test is conducted to determine
  - Short-circuit current with normal applied voltage to stator(I_{scN})
  - Power factor on short-circuit($Cos\Phi_{sc}$)
  - Total equivalent resistance referred to stator(R_{01})
  - Total equivalent reactance referred to stator(X_{01})
- Rotor is held firmly(stationary)
- Stator is connected to a variable supply
- Applied voltage is gradually increased till full load current flows in the stator and corresponding meter readings are noted.
- P_{wf} is absent since rotor is stationary
- Being very small, core loss can be neglected.
- **Power input in short-circuit test(P_{sc}) = stator copper loss + rotor copper loss.**

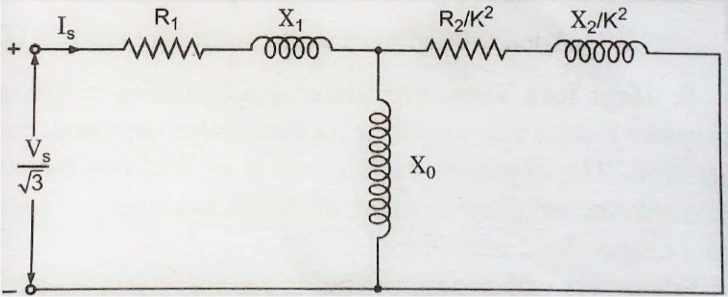

Figure 5 : Equivalent circuit diagram of induction motor under blocked rotor condition

Blocked Rotor Test

Let's consider the **stator is star connected.**

V_{sc} = Applied voltage(Line voltage), V = Normal rated voltage(Line voltage)

I_{sc} = Short-circuit current in stator winding

P_{sc} = Total input in short-circuit test

I_{scN} = Short-circuit current with normal applied voltage to stator

$$I_{scN} = I_{sc} \times \frac{V}{V_{sc}}$$

$$Power\ factor, Cos\Phi_{sc} = \frac{P_{sc}}{\sqrt{3}V_{sc}I_{sc}}$$

$$Input\ on\ short-ciruit, P_{sc} = 3I_{sc}^2R_{01}$$

$$\implies Motor\ equivalent\ resistance\ per\ phase, R_{01} = \frac{P_{sc}}{3I_{sc}^2}$$

$$Motor\ equivalent\ impedance\ per\ phase, Z_{01} = \frac{V_{sc}/\sqrt{3}}{I_{sc}}$$

$$Motor\ equivalent\ reactance\ per\ phase, X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

Blocked Rotor Test

- Usually $X_1 = X_2' = \frac{X_{01}}{2}$
- For squirrel cage induction motor, $R_2' = R_{01} - R_1$
- For slip ring induction motor, stator & rotor resistances are separated by dividing R_{01} in the ratio of dc resistances of stator & rotor windings.

Voltage Ratio Test

- Performed **only on slip ring induction motor**
- Stator winding is excited with rated voltage & frequency
- Rotor circuit is kept open circuited & standstill
- Voltmeter connected across a pair of stator terminals $\rightarrow$ Applied voltage
- Voltmeter connected across a pair of rotor terminals $\rightarrow$ Induced voltage in the rotor.
- Voltage ratio is determined from the readings of voltmeters.
- At standstill, $f_r = \mathrm{f}\ (\because \mathrm{s} = 1)$

Heat Run Test

- To find the **actual maximum temperature** attained while motor is operating under certain load condition.
- The temperature is measured both while the motor is operating & after it is shut-down
- Life of insulation of electrical equipment depends upon the temperature attained during operation.

- **In a series circuit consisting of constant source voltage, reactance & variable resistance, the locus of the current is a circle of diameter $(\frac{V}{X})$.**

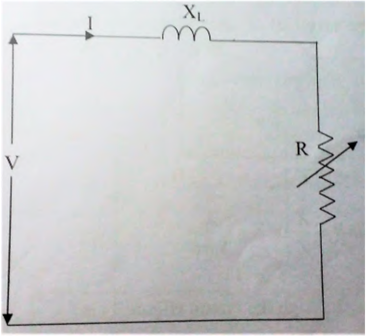

Figure 6 : Series R-L circuit

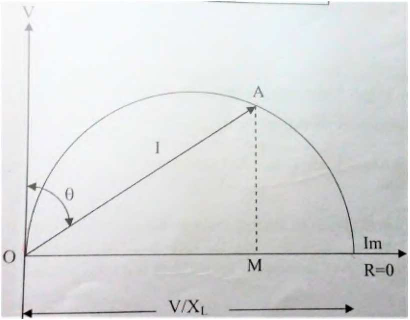

Figure 7 : Locus of current

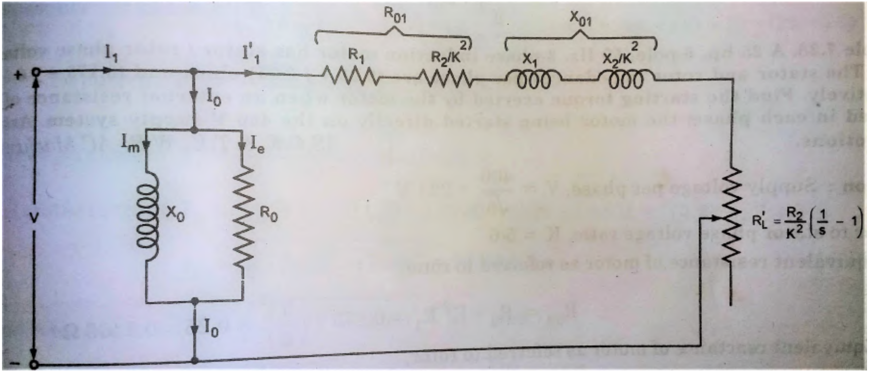

Figure 8 : Equivalent circuit of induction motor

Consider the equivalent circuit diagram of an induction motor.

- I_o is constant.
- $X_{01} = X_1 + \frac{X_2}{K^2} = X_1 + X_2' =$ Equivalent reactance referred to stator $\rightarrow$ Constant.
- $R_{01} = R_1 + \frac{R_2}{K^2} = R_1 + R_2' =$ Equivalent resistance referred to stator $\rightarrow$ Constant.
- $R_L' = R_2' \left(\frac{1-s}{s}\right) =$ Load resistance $\rightarrow$ varies with load on motor($\because$ slip varies).

$\Longrightarrow$ **Locus of I_1' is a circle of diameter $\frac{V}{X_{01}}$**

- Total current $\overrightarrow{I_1} = \overrightarrow{I_0} + \overrightarrow{I_1'} \Longrightarrow$ **Locus of I_1 is a circle.**

$\therefore$ Operating characteristics of an induction motor can be computed by use of a circle diagram.

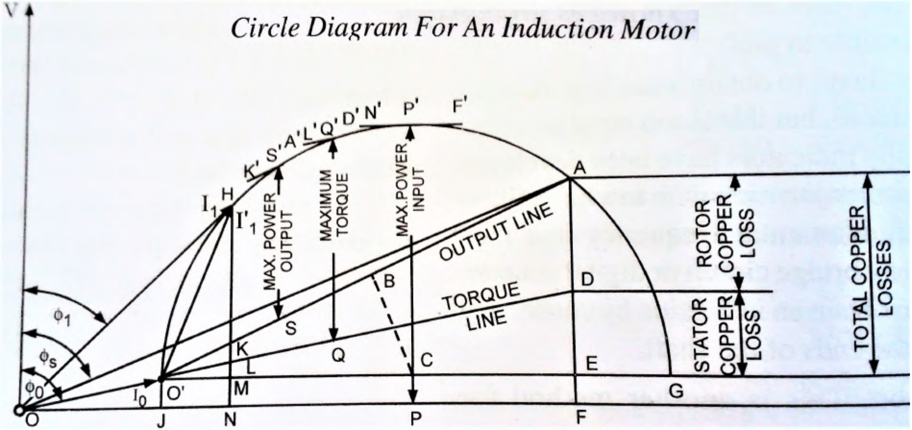

Figure 9 : Circle diagram

- Determine the circuit parameters from no-load & blocked rotor tests.

From no-load test

- Applied line voltage to stator(V_o), Line current(I_o), Total power input(P_o)

Determine

- No-load power factor, $Cos\Phi_o$
- No-load power factor angle, Φ_o

From blocked rotor test

- Applied line voltage to stator(V_{sc}), Line current(I_{sc}), Total power input(P_{sc})

Determine

- Short-circuit current corresponding to normal voltage applied to stator(I_{scN})
- Short-circuit phase angle(Φ_{sc})

Calculate,

1. No load power factor angle(Φ_o) - From no-load test data

$$\Phi_o = Cos^{-1}\left(\frac{P_o}{\sqrt{3}V_o I_o}\right) degrees$$

2. Short circuit phase angle(Φ_{sc}) - From blocked rotor test data

$$\Phi_{sc} = Cos^{-1}\left(\frac{P_{sc}}{\sqrt{3}V_{sc} I_{sc}}\right) degrees$$

3. Short circuit line current if normal voltage were applied during blocked rotor test

$$I_{scN} = I_{sc} \times \left(\frac{V}{V_{sc}}\right) Amperes$$

1.2 Construction of Circle Diagram

Calculate,

1. No load power factor angle(Φ_o) - From no-load test data

$$\Phi_o = Cos^{-1}\left(\frac{P_o}{\sqrt{3}V_o I_o}\right) degrees$$

2. Short circuit phase angle(Φ_{sc}) - From blocked rotor test data

$$\Phi_{sc} = Cos^{-1}\left(\frac{P_{sc}}{\sqrt{3}V_{sc} I_{sc}}\right) degrees$$

3. Short circuit line current if normal voltage were applied during blocked rotor test

$$I_{scN} = I_{sc} \times \left(\frac{V}{V_{sc}}\right) Amperes$$

Calculate,

1. No load power factor angle(Φ_o) - From no-load test data

$$\Phi_o = Cos^{-1}\left(\frac{P_o}{\sqrt{3}V_o I_o}\right) degrees$$

2. Short circuit phase angle(Φ_{sc}) - From blocked rotor test data

$$\Phi_{sc} = Cos^{-1}\left(\frac{P_{sc}}{\sqrt{3}V_{sc} I_{sc}}\right) degrees$$

3. Short circuit line current if normal voltage were applied during blocked rotor test

$$I_{scN} = I_{sc} \times \left(\frac{V}{V_{sc}}\right) Amperes$$

4. Power input if normal voltage were applied during blocked rotor test

$$P_{scN} = P_{sc} \times \left(\frac{V}{V_{sc}}\right)^2 watts$$

To draw Circle Diagram

- Select suitable current scale (eg. 5A/cm)
- Voltage phasor is taken along y-axis
- Draw phasor **OO'** at an angle Φ_o from phasor **OV**(lagging). The length of phasor **OO'** can be calculated from I_o using current scale
- Phasor **OO'** represents the no-load line current I_o
- Line **OF** is drawn perpendicular to voltage phasor **OV**

4. Power input if normal voltage were applied during blocked rotor test

$$P_{scN} = P_{sc} \times \left(\frac{V}{V_{sc}}\right)^2 \, watts$$

To draw Circle Diagram

- Select suitable current scale (eg. 5A/cm)
- Voltage phasor is taken along y-axis
- Draw phasor **OO'** at an angle Φ_o from phasor **OV**(lagging). The length of phasor **OO'** can be calculated from I_o using current scale
- Phasor **OO'** represents the no-load line current I_o
- Line **OF** is drawn perpendicular to voltage phasor **OV**

- Draw phasor **OA** at an angle Φ_{sc} from phasor **OV**(lagging). The length of phasor **OA** can be calculated from I_{scN} using current scale.
- Draw line **O'G** parallel to **OF**
- Join points **O'** and **A** to form line **O'A**
- Draw the perpendicular bisector of line **O'A** $\rightarrow$ ie, line **BC**
- Line **BC** intersects **O'A** at point **B** and **O'G** at point **C**
- Draw the semi-circle **O'HP'AG** with point **C** as center and **O'C** as radius
- Draw line **AF** perpendicular to **O'G** and mark points **E** and **F**
- Divide line **AE** into **AD** and **DE** such that **AD : DE = R2' : R1**

Power scale

$$P_{scN}(\text{in watts}) = \text{Line } \mathbf{AF}(\text{in cm}) \rightarrow \text{Find Power scale} = \frac{P_{scN}}{AF} \text{ W/cm}$$

- Draw the line **O'D** $\rightarrow$ **Torque line** or **Rotor input line**
- Line **O'A** $\rightarrow$ **Output line** or **Mechanical power developed line**
- Draw line **N'P'F'** parallel to line **NPF** and tangential to the semi-circle. Now **P'P** $\rightarrow$ **Maximum power input to the motor**($\rightarrow$ **P'P**(in cm) power scale(in W/cm) = Max. power input to the motor in watts)
- Draw line **L'Q'D'** parallel to torque line **O'D** and tangential to the semi-circle. Now **Q'Q** $\rightarrow$ **Maximum torque**($\rightarrow$ **Q'Q**(in cm) power scale(in W/cm) = Max. torque in synchronous watts)
- Draw line **K'S'A'** parallel to output line **O'A** and tangential to the semi-circle. Now **S'S** $\rightarrow$ **Maximum power output**($\rightarrow$ **S'S**(in cm) power scale(in W/cm) = Max. power output in watts)

1.2 Construction of Circle Diagram

- **O'J**$\rightarrow$ active component of no-load current(use current scale)
- **O'J**$\rightarrow$ power input at no-load(use power scale)
- **AF** $\rightarrow$ Active component of short-circuit current I_{scN}(with rated voltage applied on blocked rotor test) - use current scale
- **AF** $\rightarrow$ Power input on short circuit test P_{scN}(with rated voltage applied on blocked rotor test) - use power scale

At any load current(I_1)

- Mark point **H** on the semi-circle and draw the line **OH**. Line **OH** represents any load current(I_1) whose magnitude can be calculated using current scale. Also, corresponding power factor angle($Cos\Phi_o$) can be found out.

$$Motor\ input = \sqrt{3} \times V \times NH(in\ cm) \times Current\ scale(in\ A/cm)$$

$$Constant\ Losses = \sqrt{3} \times V \times NM(in\ cm) \times Current\ scale(inA/cm)$$

$$Stator\ copper\ Loss = \sqrt{3} \times V \times ML(in\ cm) \times Current\ scale(in\ A/cm)$$

$$Rotor\ input\ power = \sqrt{3} \times V \times LH(in\ cm) \times Current\ scale(in\ A/cm)$$

$$Rotor\ copper\ Loss = \sqrt{3} \times V \times LK(in\ cm) \times Current\ scale(in\ A/cm)$$

$$Motor\ output = \sqrt{3} \times V \times HK(in\ cm) \times Current\ scale(in\ A/cm)$$

$$Efficiency = \frac{Output}{Input} = \frac{\sqrt{3} \times V \times KH}{\sqrt{3} \times V \times NH} = \frac{KH}{NH}$$

$$Slip,\ s = \frac{Rotor\ copper\ loss}{Rotor\ input} = \frac{\sqrt{3} \times V \times LK}{\sqrt{3} \times V \times LH} = \frac{LK}{LH}$$

$$Power\ factor,\ Cos\Phi_1 = \frac{NH}{OH}$$

Synchronous Watt

- The torque which develops a power of 1 watt at synchronous speed of the motor

1.3 Cogging

- With certain relationships between number of poles and of stator & rotor slots in cage rotor motors, peculiar behaviour may be observed when the machine is started.
- If the number of slots on the stator & rotor are equal, **machine may refuse to start at all.** This phenomenon is known as **Cogging**.
- It is due to the heavy magnetic pull between the stator & rotor teeth.
- Starting torque developed by the machine may be insufficient to overcome the radial magnetic pull. Hence the machine may not start.
- Cogging is overcome by having stator or rotor slots prime with respect to each other(ie, no common factor)
- Example:

$$\text{Number of stator slots} = 24$$
$$\text{Number of rotor slots} = 41$$

1.3 Crawling

- The tendency of the motor to **run stably at a subnormal speed** (eg. $(1/7)^{th}$ of normal speed) with a low pitched howling sound is called **crawling.**
- In the torque-speed characteristics of induction motor(figure), reverse 5^{th} harmonic torque & forward 7^{th} harmonic torque are superimposed on the fundamental torque.
- $(T_{developed} - T_{load}) \rightarrow$ accelerates the motor.
- If motor developed torque follows fundamental torque curve, motor will accelerate to the point 'K' as a final operating point for the given load.
- Any harmonic which causes a dip in the torque curve will cause motor to accelerate to point 'M' at which developed torque is equal to load torque.
- The machine may continue to operate at point 'M' corresponding to a sub-normal operating speed.

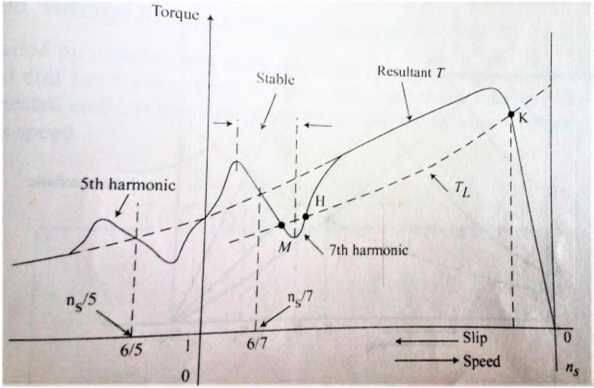

Figure 10 : Torque slip curve

- **Proper choice of coil pitch & distribution of coils while designing the winding → reduces harmonics in the air gap flux → sinusoidal flux distribution → elimination of crawling.**

1.3 Vibration & Noise

- Usually it is assumed that the flux in the air gap is distributed sinusoidally $\rightarrow$ ie, **rotating flux has sinusoidal distribution**
- In practice, it is not purely sinusoidal owing to the presence of harmonics.
- Space harmonic fields are produced due to slotting, magnetic saturation, air gap length irregularity etc. $\rightarrow$ induces harmonic voltages $\rightarrow$ harmonic current $\rightarrow$ harmonic torques $\rightarrow$ vibration & noise.

- Squirrel cage induction motor $\rightarrow$ poor starting torque($\because$ low rotor resistance)
- If rotor were made of high resistance $\rightarrow$ high starting torque $\rightarrow$ but rotor copper loss increases $\rightarrow$ efficiency decreases $\rightarrow$ hence not desirable.

Double Cage Induction Motor

- **Increased starting torque which employ change of rotor circuit impedance with change in rotor frequency** $\rightarrow$ high starting torque $\rightarrow$ high efficiency
- **High torque cage motor**
- High resistance rotor circuit during starting
- Low resistance rotor circuit during normal running condition
- $T_{st} \approx 200\%$ to 250% of full load torque.
- Starting current $\approx 400\%$ to 600% full load current.

2. Double Cage Induction Motor

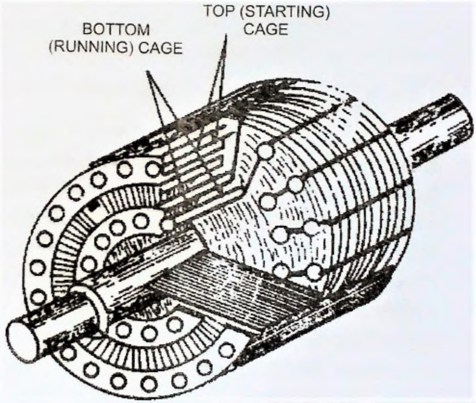

Figure 11 : Double cage induction motor

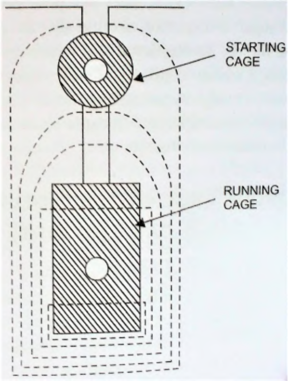

Figure 12 : Double cage induction motor

2. Double Cage Induction Motor

- Rotor $\rightarrow$ **two squirrel cage windings embedded in two rows of slots.**
- **Outer slots** $\rightarrow$ **High resistance & low leakage reactance cage** $\rightarrow$ lesser cross section $\rightarrow$ **(Manganese bars).**
- **Inner slots** $\rightarrow$ **Low resistance & high leakage reactance cage** $\rightarrow$ greater cross section $\rightarrow$ **(Copper bars)**.
- **At starting,** slip, s $\approx 1 \implies f_r = \text{f} \implies$ Leakage reactance of inner cage is high $\rightarrow$ hence,
  - Impedance of inner cage >> impedance of outer cage $\implies$ **bulk of rotor current flows through outer high resistance cage.**
  - Phase difference between the induced emf & current in inner cage will be large($\because$ high reactance). Hence torque will be small.
  - Phase difference between the induced emf & current in outer cage will be small($\because$ low reactance). Hence torque will be high.

- **At normal speed,** slip is very low $\rightarrow$ reactance of both inner & outer cages will be low($\because$ rotor reactance $\propto f_r \propto$ slip).
  - reactance $\rightarrow$ minor importance
  - **Inner cage resistance is low** $\rightarrow$ **carries bulk of the current** $\rightarrow$ inner cage provides greater proportion of torque.
  - Outer cage resistance is high $\rightarrow$ carries less current $\rightarrow$ provides less torque.
- **Outer cage** $\rightarrow$ **produces high starting & accelerating torque.**
- **Inner cage** $\rightarrow$ **provides running torque at good efficiency.**

2.1 Torque-Speed Curve

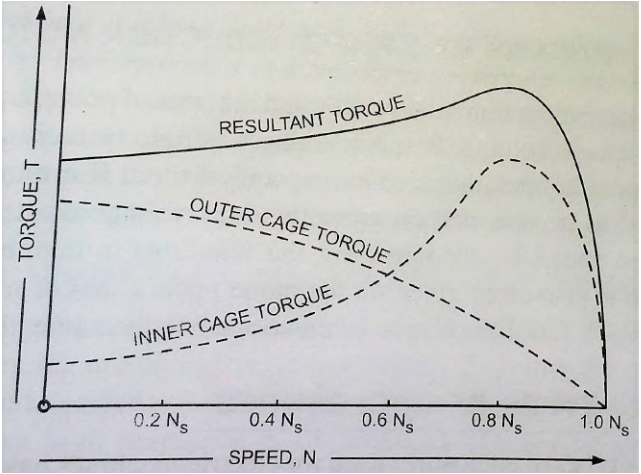

Figure 13 : Torque-Slip Curve

- Outer cage torque, inner cage torque & resultant torque.
- Desired T-N curve can be obtained by modifying individual cage resistances & leakage reactances.
  - To change resistance $\rightarrow$ change areas of cross-section of bars
  - To change leakage reactance $\rightarrow$ change width of slot opening & depth of inner cage.
- Pull out torque of double cage induction motor is less than that of normal squirrel cage induction motor because,
  - In double cage induction motor, two cages produce maximum torque at different speeds
  - Additional reactance of inner cage reduces full load power factor.

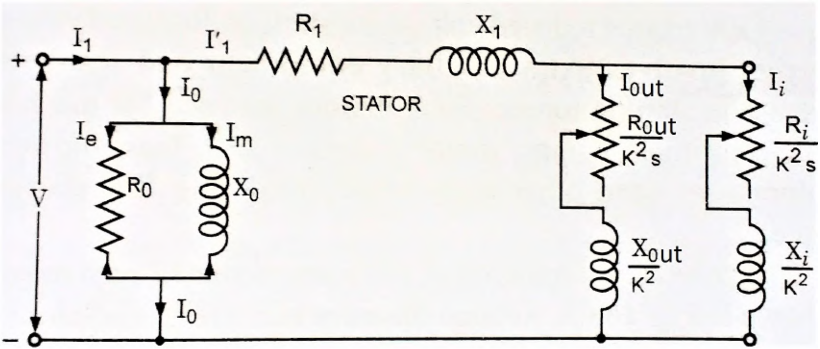

Figure 14 : Equivalent circuit of double cage induction motor

- $R_1 \rightarrow$ Stator winding resistance per phase
- $X_1 \rightarrow$ Stator winding reactance per phase
- $\frac{R_{out}}{K^2 s} \rightarrow$ Outer cage resistance referred to stator
- $\frac{X_{out}}{K^2 s} \rightarrow$ Outer cage reactance referred to stator
- $\frac{R_i}{K^2 s} \rightarrow$ Inner cage resistance referred to stator
- $\frac{X_i}{K^2 s} \rightarrow$ Inner cage reactance referred to stator

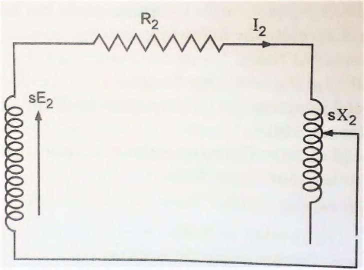

Figure 15 : Circuit diagram of induction motor rotor

- At start(ie, rotor is at standstill), $\rightarrow$ slip $= 1$ $\rightarrow$ squirrel cage **rotor is like a short-circuited secondary**.
- Rotor induced emf would be high $\rightarrow$ rotor current would be very high $\rightarrow$ consequently stator current would also be high if rated voltage were applied at start.

- Reduction in applied voltage to the stator will decease rotor current. But starting torque would be greatly decreased.($\because$ $T_{st} \propto E_2^2$)
- Starters are required to limit starting current.

Types

1. Direct On-Line(DOL) Starter
2. Primary Resistor Starting
3. Autotransformer Starter
4. Star-Delta Starter
5. Rotor Resistance Starter

Direct On-Line(DOL) Starter

- **Full voltage starting**
- Generally used for starting
  - Squirrel cage motors of capacity upto 1.5kW
  - Double cage motors & squirrel cage motors of large capacity having large rotor resistance
- Direct switching of motors to the supply mains.

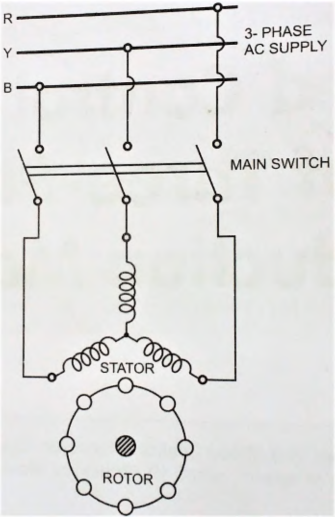

Figure 16 : Direct On-line Starting

Torque developed on starting by direct switching T = Torque developed

I_2 = rotor current per phase

R_2 = rotor resistance per phase

I_{FL} = Full load current per phase

s_{FL} = slip at full load

$$Input\ to\ rotor = T\omega$$

$$Rotor\ copper\ loss = s \times input\ power\ to\ rotor$$

$$\implies 3I_2^2R_2 = sT\omega$$

$$\therefore Torque\ developed, T = \frac{3I_2^2R_2}{s\omega} \implies T \propto \frac{I_2^2}{s}$$

$$But\ I_2 \propto I_1 \implies T \propto \frac{I_1^2}{s}$$

$$\implies T = K\frac{I_1^2}{s}, \qquad where\ K = constant$$

Direct On-Line(DOL) Starter

$$\begin{aligned}
At\ starting, s &= 1\\
Starting\ torque, T_{st} &= KI_{st}^2\\
Full\ load\ torque, T_{FL} &= \frac{KI_{FL}^2}{s_{FL}}\\
\therefore \frac{T_{st}}{T_{FL}} &= \left(\frac{I_{st}}{I_{FL}}\right)^2 \times s_{FL}\\
\implies T_{st} &= T_{FL} \times \left(\frac{I_{st}}{I_{FL}}\right)^2 \times s_{FL}
\end{aligned}$$

- When motor is connected directly across supply mains $\rightarrow I_{st} = I_{scN}$

$$\therefore Starting\ torque, T_{st} = T_{FL} \times \left(\frac{I_{scN}}{I_{FL}}\right)^2 \times s_{FL}$$

Primary Resistor Starter

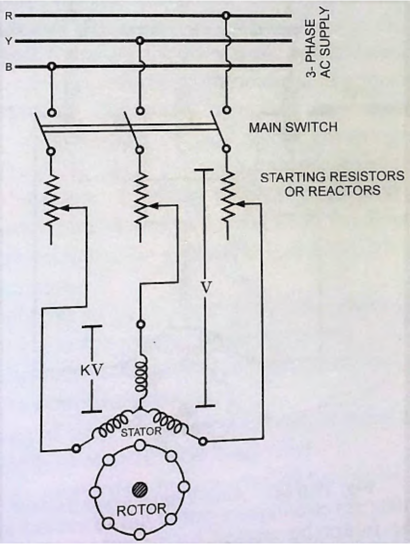

Figure 17 : Primary Resistor Starter

Primary Resistor Starter

- Reduced voltage to stator is achieved by connecting resistor in series with each stator lead during the starting period.
- As motor picks up speed, resistors are cut-out in steps & finally short circuited when motor attains normal operating speed.
- If normal supply voltage is V & by using line resistance starter, the voltage is reduced to KV.
- Starting current is also reduced to KI_{scN}

At starting,

$$
\begin{aligned}
Starting\ torque,\ T_{st} &= T_{FL}\left(\frac{I_{st}}{I_{FL}}\right)^2 \times s_{FL} \\
&= T_{FL}\left(\frac{KI_{scN}}{I_{FL}}\right)^2 \times s_{FL} \\
&= K^2 \times T_{FL}\left(\frac{I_{scN}}{I_{FL}}\right)^2 \times s_{FL} \\
&= K^2 \times\ Torque\ obtained\ by\ direct\ switching
\end{aligned}
$$

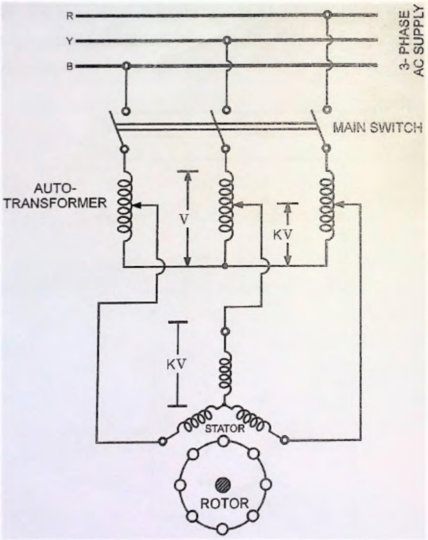

Figure 18 : Auto-transformer starter

- Reduced voltage is obtained by taking tappings at suitable points from a 3-phase auto-transformer.
- Over-load protection, Under-voltage protection, Thermal over load relays.

Torque developed on starting

Let K = Transformation ratio of auto-transformer

I_{scN} = Starting current when normal voltage is applied

$$Applied\ voltage\ to\ stator\ winding\ at\ starting = KV$$

$$\therefore Motor\ input\ current, I_{st} = KI_{scN}$$

$$\begin{aligned} Supply\ current &= 1^o\ current\ of\ autotransformer \\ &= K \times 2^o\ current\ of\ autotransformer \\ &= K^2 I_{scN} \end{aligned}$$

At starting,

$$
\begin{aligned}
\textit{Motor input current}, I_{st} &= KI_{scN} \\
\textit{Starting torque}, T_{st} &= T_{FL}\left(\frac{I_{st}}{I_{FL}}\right)^2 \times s_{FL} \\
&= T_{FL}\left(\frac{KI_{scN}}{I_{FL}}\right)^2 \times s_{FL} \\
&= K^2 \times T_{FL}\left(\frac{I_{scN}}{I_{FL}}\right)^2 \times s_{FL} \\
&= K^2 \times \textit{ Torque obtained by direct switching}
\end{aligned}
$$

- **Line current & starting torque are reduced in square ratio.**

Advantages

- Voltage is reduced by transformation & not by dropping voltage in resistors(as in case of primary resistance starting) $\rightarrow$ current & power drawn from the supply are reduced.
- Internal losses of the starter itself is low.
- Highest starting torque per ampere of supply current.
- Adjustment of starting voltage by selection of proper tap on autotransformer.
- Suitability of long starting torque.
- Motor current larger than supply current.
- Can be employed for star connected as well as delta connected motors.

Drawbacks

- Low power factor
- Higher cost in case of lower output rating motors.

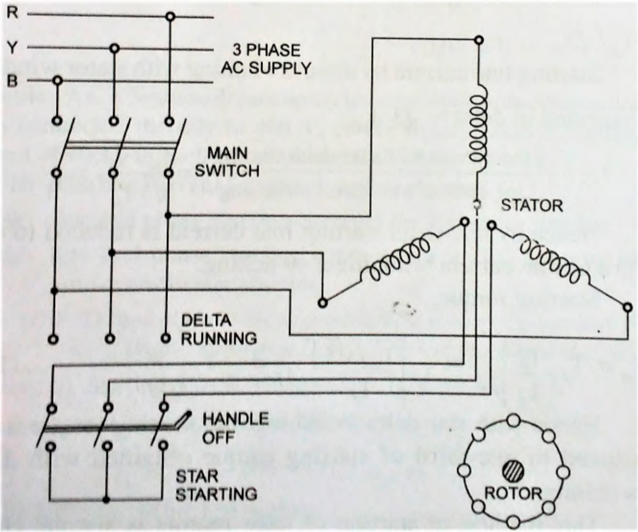

Figure 19 : Star-Delta Starter

- Principle - **Voltage across each phase winding of a 3-phase star connected winding is 57.7%(ie, $1/\sqrt{3}$) of the line voltage** whereas same winding connected in delta will have full line voltage across each phase windings.
- Star-Delta starter $\rightarrow$ connects stator winding in star across the supply at starting instant.
- As the motor attains speed $\rightarrow$ stator windings are reconnected in delta through a change-over switch across the same supply voltage.
- At starting $\rightarrow$ star connected stator $\rightarrow$ $V_{ph} = \frac{V_L}{\sqrt{3}}$

  Hence starting current per phase, $I_{st} = \frac{I_{scN}}{\sqrt{3}}$(same as line current)
- Starting line current by DOL switching with stator windings connected in delta $= \sqrt{3}I_{scN}$.

$$\therefore \frac{Line\ current\ with\ star-delta\ starting}{Line\ current\ with\ direct\ switching} = \frac{I_{scN}/\sqrt{3}}{\sqrt{3}I_{scN}} = \frac{1}{3}$$

- **By star-delta starting, line current is reduced to one-third of line current with DOL switching.**
- Starting torque,

$$
\begin{aligned}
T_{st} &= T_{FL} \times \left(\frac{I_{st}}{I_{FL}}\right)^2 \times s_{FL} \\
&= T_{FL} \times \left(\frac{I_{scN}/\sqrt{3}}{I_{FL}}\right)^2 \times s_{FL} \\
&= \frac{1}{3} \times T_{FL} \left(\frac{I_{scN}}{I_{FL}}\right)^2 \times s_{FL}
\end{aligned}
$$

- $\Longrightarrow$ Starting torque(T_{st}) is **reduced to one-third** of starting torque obtained with DOL switching.

Advantages

- Simple
- Cheap
- Efficient ($\because$ no power is lost as in case of primary resistance starting)
- Suitable for high inertia & long acceleration loads.

Drawbacks

- Motor needs to be delta connected for normal operation. Hence 6 terminals of the stator winding are to be brought out.
- Reduction in voltage is fixed.
- Starting torque is low.
- Limited to application where high starting torque is not required.
- Unsuitable for line voltage exceeding 3000V as excess stator turns required for delta connection.

Rotor Resistance Starter

- Starting of **slip-ring induction motor**
- Full line voltage is applied across stator
- Variable resistance is added in each phase of rotor circuit.
- Adding external resistance to the rotor circuit $\rightarrow$
  - Reduces starting current
  - Increases starting torque
  - Improves power factor
- At starting moment $\rightarrow$ Complete external resistance is included in the rotor circuit.
- As motor accelerates $\rightarrow$ external resistance is cut-out in steps.
- When motor runs at normal speed $\rightarrow$ rotor winding is short-circuited through slip-rings.

Rotor Resistance Starter

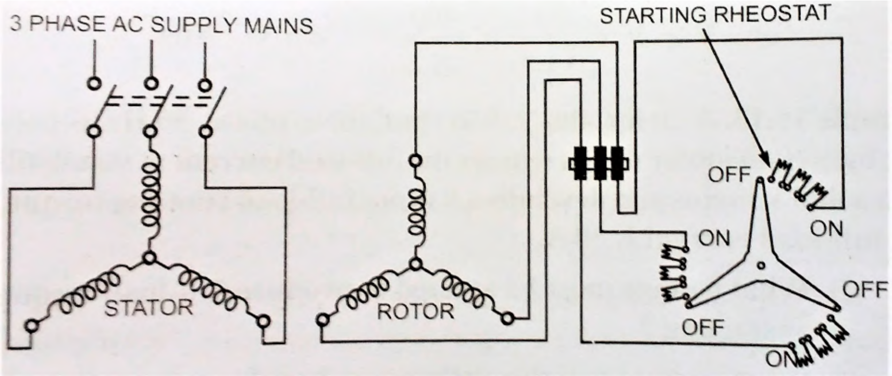

Figure 20 : Rotor Resistance Starter

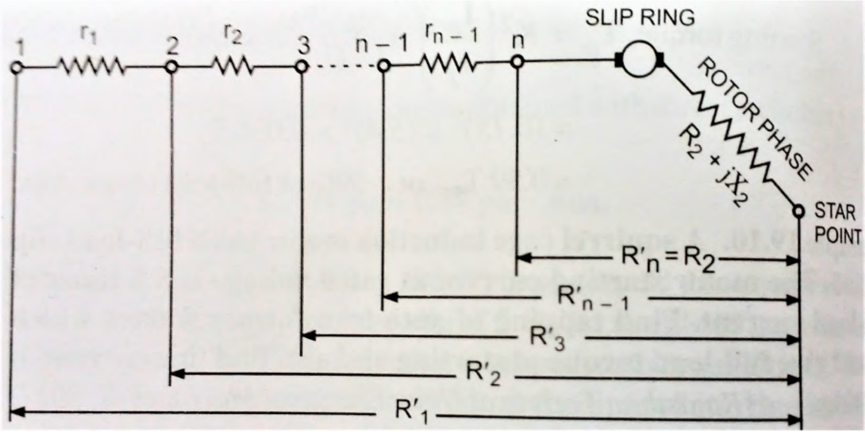

Figure 21 : Rotor resistance starter(Circuit)

Calculation of Resistance steps

- Consider one phase of rotor with R_2 & X_2
- $r_1,\ r_2, \ldots\ldots r_{n-1} \rightarrow$ resistances of different steps in each phase.
- $R'_1,\ R'_2, \ldots\ldots R'_{n-1}$ & $R'_n \rightarrow$ total resistance per phase in rotor circuit on the 1^{st}, 2^{nd},.......$(n-1)^{th}$ & n^{th} stud.
- Assumption : *Motor starts against a constant torque & rotor current fluctuates between two fixed values of I_{2max} & I_{2min} respectively.*
- At the commencement of each step, current $= I_{2max}$ & rotor resistance in R'_1 for slip s_1, R'_2 for slip s_2 and so on.

$$I_{2max} = \frac{E_2}{\sqrt{\left(\frac{R'_1}{s_1}\right)^2 + X_2^2}} = \frac{E_2}{\sqrt{\left(\frac{R'_2}{s_2}\right)^2 + X_2^2}} = \frac{E_2}{\sqrt{\left(\frac{R'_3}{s_3}\right)^2 + X_2^2}} = \ldots.$$

$$\Longrightarrow \frac{R'_1}{s_1} = \frac{R'_2}{s_2} = \frac{R'_3}{s_3} = \ldots\ldots\ldots\ldots = \frac{R'_{n-1}}{s_{n-1}} = \frac{R_2}{s_{max}}$$

where $s_{max} =$ slip under normal operating condition.

- At starting $\rightarrow$ knob is on 1^{st} stud $\rightarrow$ rotor resistance $= R_1'$ $\rightarrow$ slip, $s_1 = 1 \rightarrow$ rotor current per phase $= I_{2max}$.
- As motor gains speed, current decreases till current becomes I_{2min} at slip s_2.

$$\therefore I_{2min} = \frac{E_2}{\sqrt{\left(\frac{R_1'}{s_2}\right)^2 + X_2^2}} = \frac{E_2}{\sqrt{\left(\frac{R_2'}{s_3}\right)^2 + X_2^2}} = \frac{E_2}{\sqrt{\left(\frac{R_3'}{s_4}\right)^2 + X_2^2}} = \ldots.$$

$$\implies \frac{R_1'}{s_2} = \frac{R_2'}{s_3} = \frac{R_3'}{s_4} \ldots\ldots\ldots\ldots = \frac{R_{n-1}'}{s_{max}}$$

$$\therefore \frac{s_2}{s_1} = \frac{s_3}{s_2} = \frac{s_4}{s_3} = \ldots\ldots\ldots = \frac{s_{max}}{s_{n-1}} = \frac{R_2'}{R_1'} = \frac{R_3'}{R_2'} = \frac{R_4'}{R_3'} = \ldots\ldots\ldots\ldots = \frac{R_2}{R_{n-1}'} = K$$

Rotor Resistance Starter

- Total resistance in the rotor circuit per phase on 1_{st} stud,

$$R_1' = R_2 \times \frac{s_1}{s_{max}} = \frac{R_2}{s_{max}}$$
$$R_2' = KR_1'$$
$$R_3' = KR_2' = K^2 R_1'$$
$$R_2 = KR_{n-1}' = K^{n-1} R_1' = K^{n-1} \frac{R_2}{s_{max}}$$
$$\implies K = (s_{max})^{\frac{1}{n-1}}$$

where (n-1) = number of sections in starter.

- Resistances of various sections,

$$r_1 = R_1' - R_2' = R_1'(1 - K)$$
$$r_2 = R_2' - R_3' = R_1'(K - K^2) = Kr_1$$
$$r_3 = R_3' - R_4' = R_1'(K^2 - K^3) = Kr_2 = K^2 r_1$$

4. Braking of Induction Motor

- For stopping induction motor $\rightarrow$ switch-off supply $\rightarrow$ motor will come to rest slowly.
- For rapid stopping $\rightarrow$ use either **mechanical or electrical braking.**
- Electrical braking $\rightarrow$ provides precise control & smoothness of operation.
- **In electrical braking, direction of developed torque will be in opposite to that of rotation.**

Electrical Braking

1. Plugging or Counter-current Braking
2. Dynamic Braking or Rheostatic Braking
3. Regenerative Braking

Plugging or Counter-current Braking

- Plugging of induction motor is achieved by **reversing any two of the three phases** $\rightarrow$ reverses the direction of rotating magnetic field.
- At the instant of reversal, motor runs in opposite direction to that of field $\rightarrow$ relative speed is approximately twice(ie, (2-s) times) of synchronous speed $\rightarrow$ slip = (2-s) $\implies$ **induced voltage in rotor will be twice** of normally induced voltage at standstill $\rightarrow$ **additional insulation** must be provided to withstand this high voltage.
- During plugging $\rightarrow$ motor acts as a brake $\rightarrow$ P_{mech} is dissipated as heat, rotor copper loss is dissipated as heat & additional iron losses are also produced since f_r is high $\rightarrow$ Heat developed in rotor during braking period is about 3 times the heat developed during starting period.

Plugging or Counter-current Braking

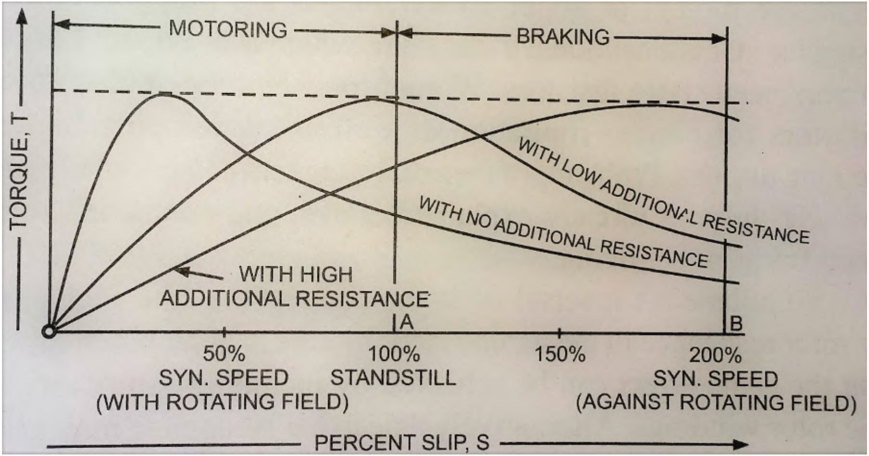

Figure 22 : Torque-Slip Curve

Plugging or Counter-current Braking

- The ordinate at point **B** $\rightarrow$ Torque at the instant of plugging.
- From point B $\rightarrow$ Torque increases gradually as the motor approaches standstill point.
- It is **essential to disconnect the supply to motor when motor comes to zero speed.** Otherwise motor will start rotating in the opposite direction.
- Addition of resistance in the rotor circuit gives more braking torque(stator current can be minimised). Hence **wound rotor(or slip-ring) induction motors are more suitable for plugging.**

Dynamic Braking or Rheostatic Braking

- Dynamic Braking or Rheostatic Braking $\rightarrow$ **Disconnect stator winding from AC supply & excite it from a DC source to produce a stationary field.**
- Stator winding is employed as DC field winding & rotor winding as an armature winding.
- In slip ring motor $\rightarrow$ external resistances can be inserted to rotor circuit to provide load.
- In squirrel cage motor $\rightarrow$ rotor circuit itself acts as a load.
- DC excitation can be provided either by an independent DC source or from AC mains through transformer-rectifier set.
- Fig. (a) & (b) are commonly used.
- Fig. (c) & (f) $\rightarrow$ **uniform current loading** of all the three phases.

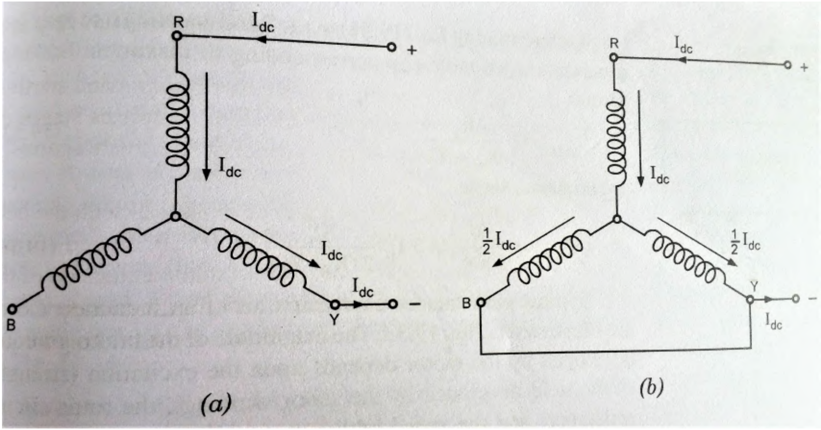

Figure 23 : Connection diagram

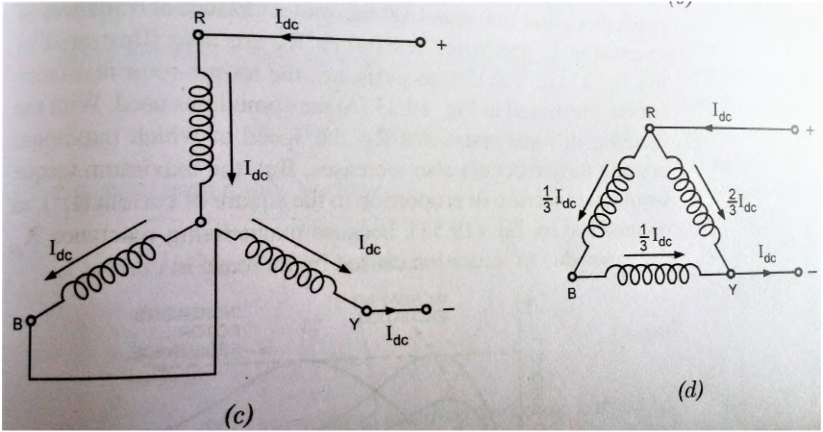

Figure 24 : Connection diagram

Dynamic Braking or Rheostatic Braking

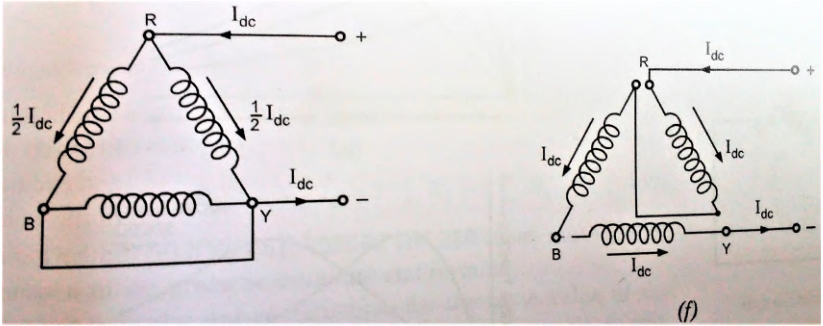

Figure 25 : Connection diagram

Dynamic Braking or Rheostatic Braking

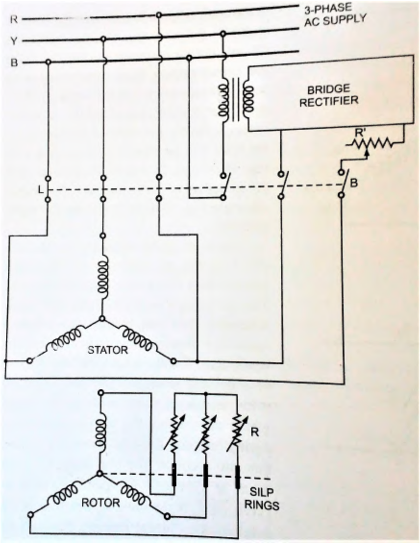

Figure 26 : Connection diagram

Dynamic Braking or Rheostatic Braking

- Machine operates as motor with contactors **L** closed.
- Open **L** & close **B** → Dynamic braking → DC is supplied through two stator phases, the third being left open circuited(refer figure).
- Resistance(R') is inserted in stator circuit to limit stator current.
- In case of slip-ring induction motor → Resistance(R) is inserted in the rotor circuit to control the braking effect.
- During normal operation of motor, stator magnetic field rotates at synchronous speed in the **same direction** as that of rotor, but **slightly faster** than rotor.
- DC excitation of stator winding → magnetic field which is **stationary in space.**
- Rotor conductors move past the field with a speed (1-s)N_s or sN_s → current is induced in the rotor conductors → produces braking torque.

- **Induction motor when runs at speed above N_s $\rightarrow$ slip & torque become negative $\rightarrow$ Induction generator $\rightarrow$ feeds power back to supply line.**
- To run induction motor above N_s,
  - Switching over to a low frequency supply in frequency controlled induction motors.
  - Downward motion of a loaded hoisting machine such as crane hoists, excavators etc.
  - Switching over to a large pole number of operation from a smaller one in multi-speed squirrel cage motors.
- When the speed of induction motor goes above N_s, braking operation starts automatically.
- In regenerative braking method, braking effect can only be achieved at speed above synchronous speed(hence limited application).
- Saves roughly 20% of total energy.
- Considerably reduces brake shoe wear.

- Industrial drive requires good speed characteristics.
- Speed of induction motor,

$$N = N_s \times (1 - s) = \frac{120f}{P} \times (1 - s)$$

$\implies$ Speed(N) of induction motor depends on
1) Supply frequency 2) Number of poles 3) Slip

- Speed can be controlled by the action on
  1. **Stator side**
     - **Variation of supply frequency(f)**
     - **Variation of applied voltage(V)**
     - By changing number of poles(P)
  2. **Rotor Side**
     - **By changing resistance in the rotor circuit**
     - By introducing into the rotor an emf of same frequency as the fundamental emf of rotor

- **Speed control by variation of supply voltage**
- Slip-control method with constant frequency, variable supply voltage.
- Voltage applied to the stator is varied for varying the speed.
- Torque developed by the induction motor,

$$T = \frac{K\ s\ R_2\ E_2^2}{R_2^2 + (sX_2)^2}$$

- Torque developed(T) by an induction motor $\propto$ square of induced emf in rotor(E_2)
- Induced emf in rotor(E_2) $\propto$ applied voltage(V)
- $\implies$ $\mathbf{T} \propto V^2$
- $\implies$ Reduction in applied voltage(V), decreases developed torque(T). Hence speed will be decreased for a constant load.

Stator Voltage Control

- **Variation in supply voltage doesn't cause any change in synchronous speed(N_s).**
- Stator Voltage Control is simple, cheap with low maintenance cost. But
    - Operation at voltages exceeding rated voltage is restricted by magnetic saturation
    - Large change in voltage is required for a relatively small change in speed.
    - Developed torque greatly reduces with reduction in supply voltage
    - Range of speed control is very limited(only in downward direction).

- Wide speed-control range with gradual variation of speed.
- Needs variable frequency supply $\rightarrow$ The required auxiliary equipments increase the overall cost.
- To operate induction motor at different frequencies with constant values of efficiency, power factor, slip etc, supply voltage(V) must be varied with change in frequency according to the equation,

$$\frac{V}{V_o} = \frac{f}{f_o}\sqrt{\frac{T}{T_o}}$$

  where,
  V & T are applied voltage & torque corresponding to frequency f.
  V_o & T_o are applied voltage & torque corresponding to frequency f_o.
- For constant torque, T $= T_o \implies$

$$\frac{V}{V_o} = \frac{f}{f_o}$$

- **Applied voltage to the stator must vary in proportion to the frequency**

Speed Control by Variation of Supply Frequency

- An imbalance in the (V/f) ratio can cause
  - An excess flux $\rightarrow$ saturation
  - Reduced flux $\rightarrow$ reduced torque per ampere of current
- Excessive flux created due to reduction in frequency made for the purpose of reducing speed at constant voltage will cause increase in iron losses $\rightarrow$ excessive heating
- If the ratio (V/f) is maintained, flux per pole will be constant.

Speed Control by Variation of Supply Frequency

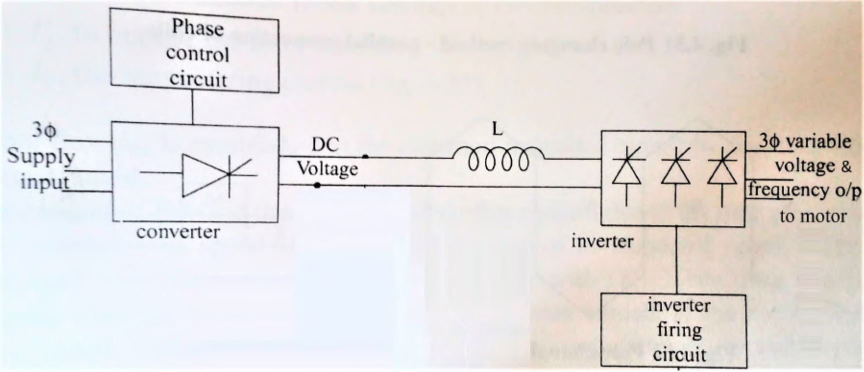

Figure 27 : Speed control of induction motor(V/f method)

Torque-Slip Characteristics with Variation of Supply Frequency

(keeping V/f ratio constant)

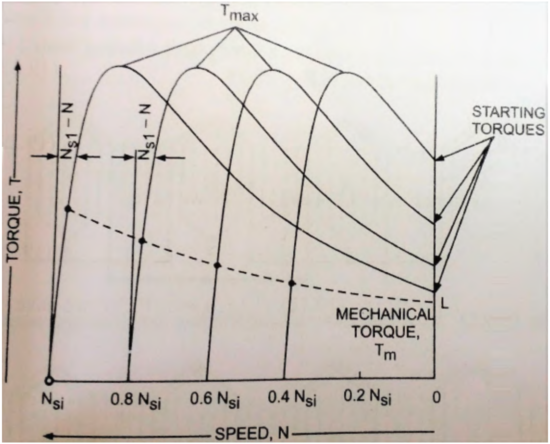

Figure 28 : Torque-Slip(speed) Curve

Torque-Slip Characteristics with Variation of Supply Frequency
(keeping V/f ratio constant)

Conclusions

- Slip corresponding to maximum torque decreases with increasing frequency.
- Maximum torque is independent of frequency.
- Starting torque decreases with increasing frequency.

Rotor Resistance Control

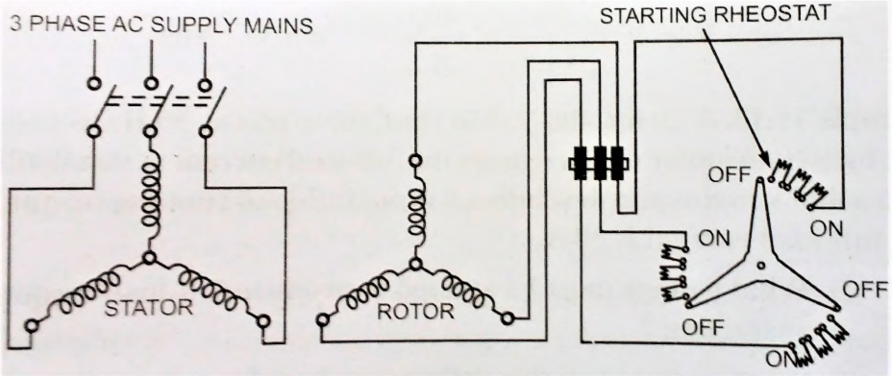

Figure 29 : Rotor Resistance Control

Rotor Resistance Control

- **Speed control by variation of rotor resistance**
- Applicable only in case of slip-ring induction motors.
- Torque developed by the induction motor,

$$T = \frac{K\ s\ R_2\ E_2^2}{R_2^2 + (sX_2)^2}$$

- When speed(N) is very near to synchronous speed(N_s), slip(s) is very low. $\rightarrow sX_2 << R_2 \implies sX_2$ can be neglected as compared to R_2.

$$\implies T \propto \frac{s}{R_2}$$

- $\implies$ For a given load, speed can be decreased by increasing rotor resistance.
- **Speed only below rated speed can be achieved.**

Q1. Draw the circle diagram of a 20hp, 400V, 50Hz, 3-phase star connected induction motor from the following test data(line values)

No load test	:	400V	9A	pf = 0.2
Blocked rotor test	:	200V	50A	pf = 0.4

From the circle diagram find
(a) Maximum power output
(b) Maximum power input
(c) Maximum torque
(d) Constant loss
(e) Starting torque

At full load,

(f) line current

(g) power factor

(h) Motor input

(i) Stator copper loss

(j) Rotor input power

(k) Rotor copper loss

(l) Motor output

(m) Efficiency

(n) Slip

(o) Speed

(p) Torque

(q) Maximum power factor

Assume that the stator & rotor copper losses are divided equally in the blocked rotor test.

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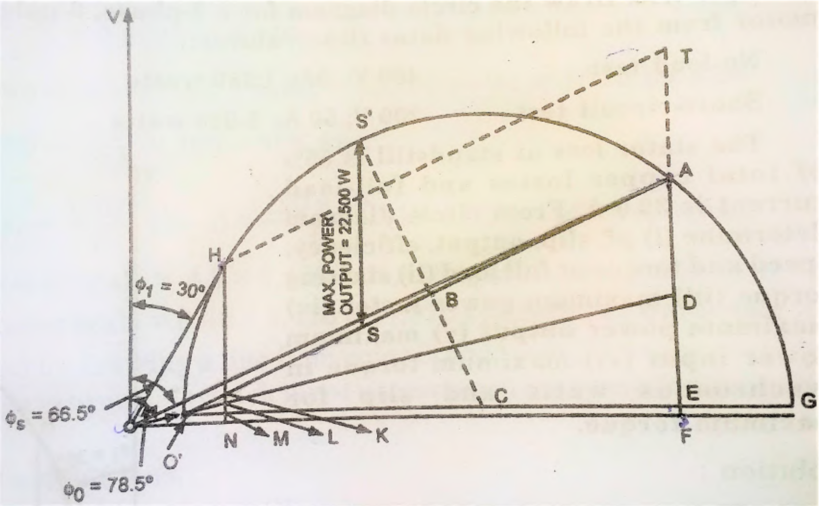

Figure 30 : Circle diagram(a)

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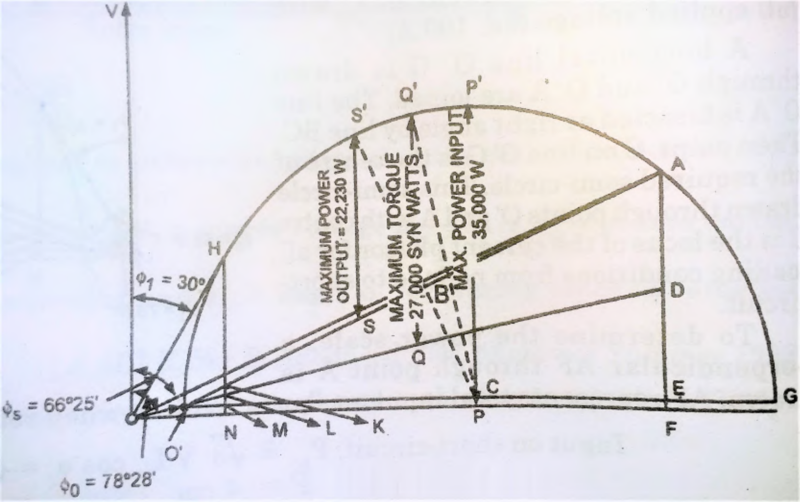

Figure 31 : Circle diagram(b)

Q2. Draw the circle diagram of a 5hp, 200V, 50Hz, 4 pole, 3-phase star connected induction motor from the following test data

No load test	:	200V	5A	350W
Blocked rotor test	:	100V	26A	1700W

From the circle diagram find
(a) Maximum power output
(b) Maximum power input
(c) Maximum torque
(d) Constant loss
(e) Starting torque

At full load,

(f) line current

(g) power factor

(h) Motor input

(i) Stator copper loss

(j) Rotor input power

(k) Rotor copper loss

(l) Motor output

(m) Efficiency

(n) Slip

(o) Speed

(p) Torque

(q) Maximum power factor

Rotor copper loss at standstill is equal to half of the total copper loss

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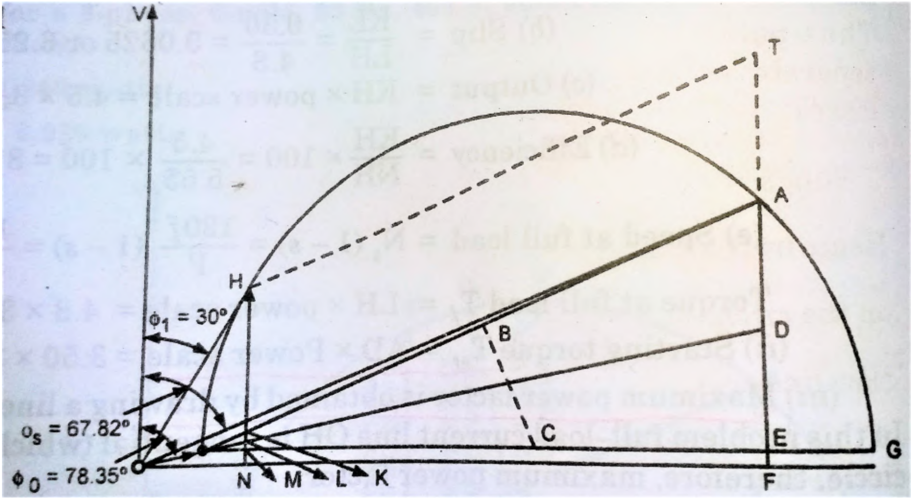

Figure 32 : Circle diagram

Q3. Calculate the reduction in the starting current & starting torque when the supply voltage to a cage rotor motor is 80% instead of 100%?

Ans:
% reduction in $I_{sc} = 20\%$
% reduction in $T_{st} = 36\%$

Q4. A 3-phase induction motor has a ratio of T_{max} to T_{FL} as 2.5:1. Determine the ratio of actual starting torque to full load torque for star-delta starting. Given $R_2 = 0.4\Omega$ & $X_2 = 4\Omega$.

Ans:

Actual T_{st} : $T_{FL} = 0.165 : 1$

Q5. A 12kW, 3-phase, 6-pole, 50Hz, 400V, delta connected induction motor runs at 960rpm on full load. If it takes 85A on direct starting, find the ratio of the starting torque to full load torque with a star-delta starter. Full load efficiency & power factor are 88% & 0.85 respectively.

Ans:

$T_{st} : T_{FL} = 0.18 : 1$

Q6. A 3-phase induction motor takes a starting current which is 5 times full load current at normal voltage. Its full load slip is 4%. What autotransformer ration would enable the motor to be started with not more than twice the full load current drawn from the supply?. What would be the starting torque under this condition.

Ans:
K = 0.6325
$T_{st} = 0.4 \times T_{FL}$

Q7. Design the five sections of a 6-stud starter for a 3-phase wound rotor induction motor. The slip at full load is 2% & starting current is 1.5 times the full load current. The resistance of rotor is 0.02Ω per phase.

Ans:

$r_1 = 0.336\Omega$
$r_2 = 0.1667\Omega$
$r_3 = 0.0826\Omega$
$r_4 = 0.041\Omega$
$r_5 = 0.0203\Omega$

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Thank You

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Synchronous and Induction Machines

Synchronous and Induction Machines (EE-202)

by

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Module 6 - Overview

1. Induction Generator
   - Principle of Operation
   - Grid Connected & Self-excited Operation
   - Comparison of Induction Generator with Alternator
2. Synchronous Induction Motor
   - Principle of Operation
3. Single-phase Induction Motor
   - Double Revolving Field Theory
   - Torque-Slip Curve
   - Equivalent Circuit
   - Types
     - Split-Phase Induction Motor
     - Capacitor-Start Induction Motor
     - Capacitor-Start and Capacitor-Run Induction Motor
     - Shaded Pole Induction Motor
   - Applications

1.1 Principle of Operation

- If an induction machine with stator connected to supply is driven at a speed higher than synchronous speed by a prime mover → rotor overtakes rotating magnetic field($N > N_s$) → slip becomes negative → rotor conductors cut magnetic field in the reverse direction to motor rotation → emf & current in rotor will reverse their direction → machine delivers electrical energy to mains → **Induction generator**.

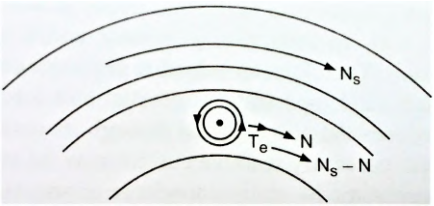

Figure 1 : Motoring Action(+ve slip)

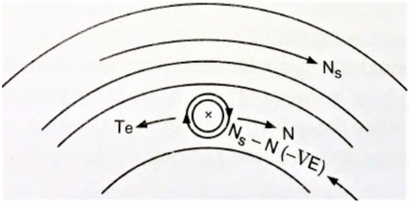

Figure 2 : Generating Action(-ve slip)

Torque-Slip curve

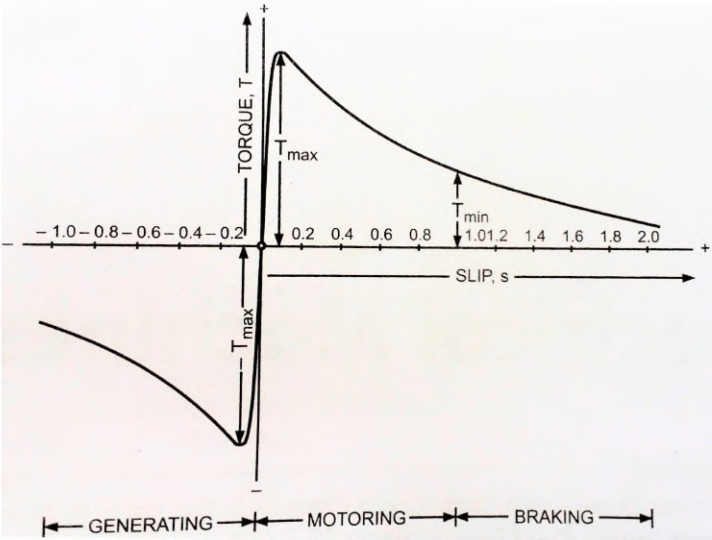

Figure 3 : Torque-Slip curve of Induction Machine

Torque-Slip curve

- The characteristic equations & equivalent circuit diagram of induction generator can be obtained from those of induction motor by introducing slip(s) with negative sign.
- Rotor emf increases in proportion to slip
- Torque, $T \propto \Phi_2 I_2 Cos\Phi_2$
- At T_{max}, s $= (R_2/X_2)$
- Induction generator supplies active current to the mains but receives lagging reactive current(magnetising current) from the mains $\implies$ Induction generator requires a source of reactive energy(eg: static capacitor bank connected in parallel with induction machine).
- Capacitor bank must be large enough to supply the reactive power normally drawn by the machine.

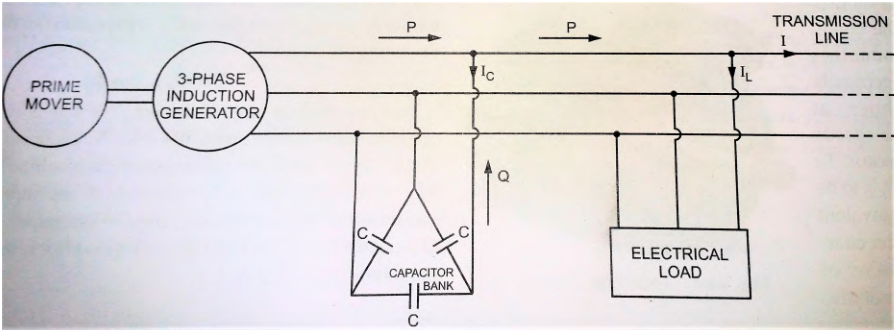

Figure 4 : Induction generator connected to mains

- Capacitor banks provide the necessary reactive power for the operation of induction generator.

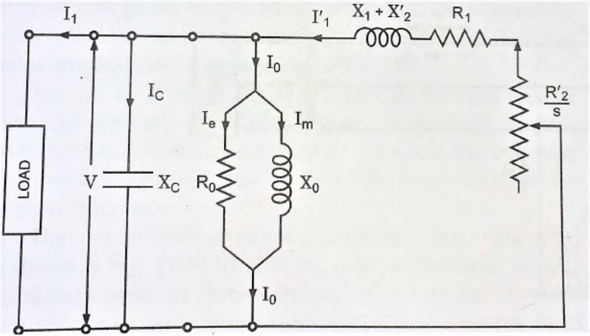

Figure 5 : Equivalent circuit of Induction Generator

- with s = -ve, load resistance $= -R_2'\left(\frac{1+s}{s}\right)$
- -ve load resistance means it no longer absorbs power but is a source of power → mechanical to electrical energy conversion.

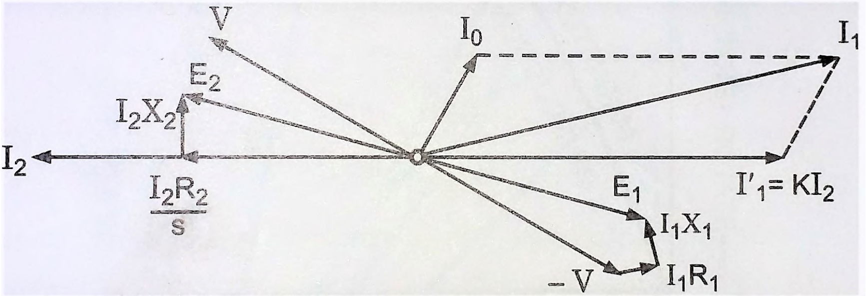

Figure 6 : Phasor diagram of Induction Generator

- In induction generator, induced emf(E_1) is the phasor sum of reversed supply voltage(-V) & impedance drop in stator winding.

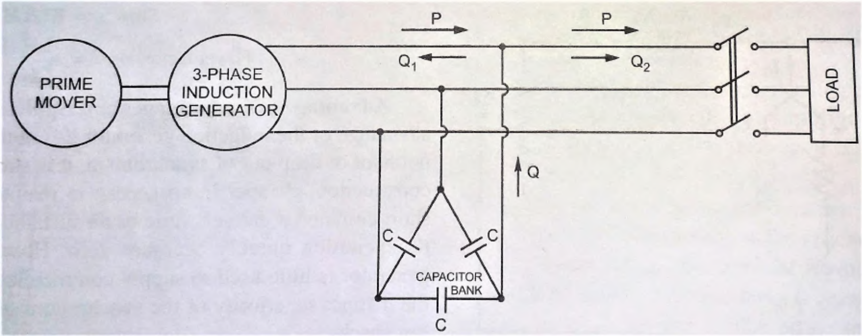

Figure 7 : Isolated Induction Generator

1.2 Self-excited or Isolated Induction Generator

- Capacitor bank provides reactive power of both induction generator as well as load.
- Terminal voltage increases with increase in capacitance 'C'.
- If the capacitance is insufficient, the generator will not build-up voltage.
- When induction generator is started, self-excitation of the machine occurs.
- Capacitance reactive power(Q) = Reactive power required by 3-phase induction generator(Q_1) + Reactive power required by the load(Q_2)

1.2 Self-excited or Isolated Induction Generator

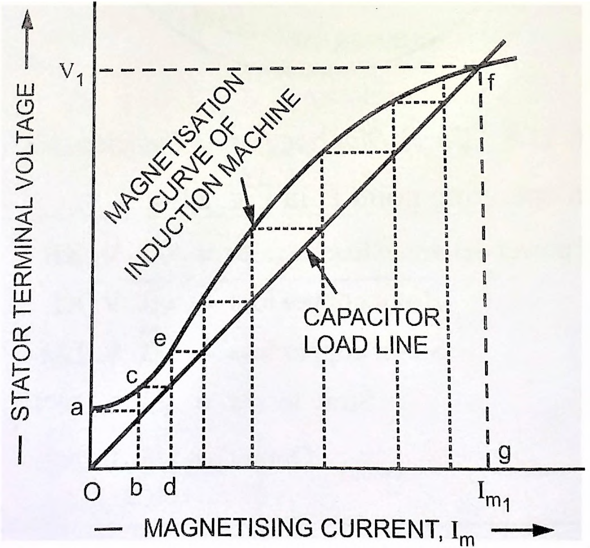

Figure 8 : Building-up Voltage in Induction Generator at No-load

Building-up Voltage in Induction Generator at No-load

- When induction generator is started & run at the required speed, the residual magnetism present in the rotor iron induces a small terminal voltage **'oa'** across the stator terminals(fig. 8).
- The voltage **'oa'** results in a capacitor current **'ob'**
- The current **'ob'** produces flux & therefore more generated voltage **'bc'** across the stator terminals
- The increased voltage **'bc'** increases capacitor current to **'od'** which induces voltage **'de'**
- This cumulative process of voltage build-up continues till capacitor load line intersects the saturation curve of induction generator at point **'f'**
- **'gf'** represents no-load induced emf
- Voltage build-up depends upon the value of capacitor. Higher the value of capacitance, more is the voltage build-up(fig. 9).

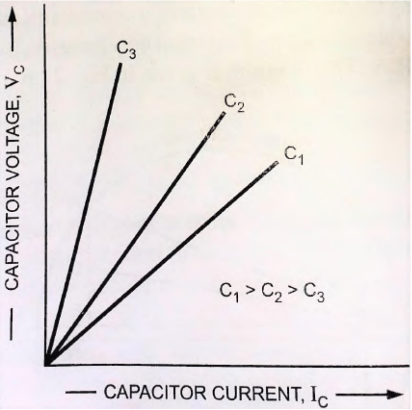

Figure 9 : No-load voltages for different capacitances

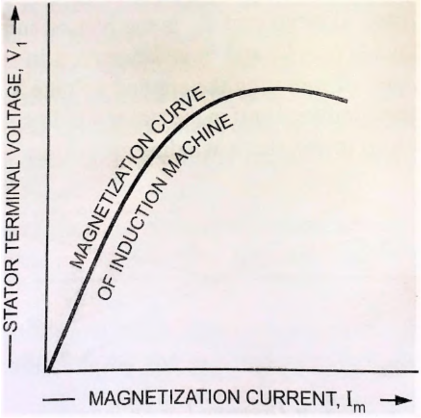

Figure 10 : Characteristic of Induction Generator

1.3 Comparison of Induction Generator with Alternator

Induction Generator

- No DC excitation is required.
- Induction generator only generates when stator is connected to the mains($\because$ magnetising current is drawn from the mains).
- Frequency & voltage of induction generator are those of the line to which it is connected irrespective of the load.
- It doesn't have definite speed for a given frequency. But speed with constant frequency varies with load.
- No synchronising is required since the machine can't generate any emf until it is connected to the supply mains.
- Power factor of the output is fixed in value by generator characteristics & is independent of external circuit.

Induction Generator

Advantages

- It doesn't drop out of synchronism
- Simple & rugged in construction
- Cheaper in cost & Easy maintenance
- When short-circuited, it delivers little or no sustained power since it's excitation becomes zero.

Disadvantages

- It can't be operated independently(self-excitation by capacitors is of limited importance).
- It can deliver only leading current
- Dangerously high voltages may occur over long transmission line if synchronous machine at far end become disconnected & the line capacitance excites the induction machine.

Applications

- Tidal power plants & Wind power stations
- Braking purposes

- Synchronous Induction Motor → capable of running both as induction motor(during starting period) & as a synchronous motor(during normal running).
- Rotor consists of three phase winding.
- It's a round rotor machine

2.1 Principle of Operation

- For synchronous mode of operation, rotor winding needs to be excited with DC supply → develops alternate N & S poles → but pole axes due to DC excitation are fixed in space → these fixed magnetic poles get magnetically locked with the rotating magnetic field developed by 3-phase stator windings carrying AC current → motor runs at a constant speed equal to synchronous speed.
- DC excitation to the rotor winding can be provided in many ways.

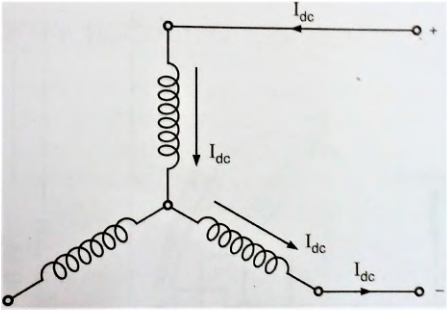

Figure 11 : Rotor circuit(a)

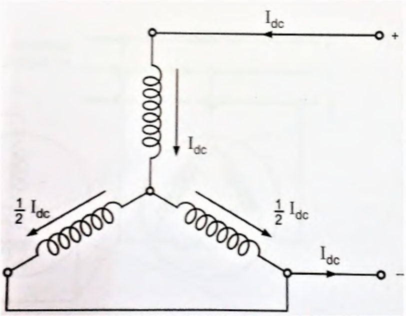

Figure 12 : Rotor circuit(b)

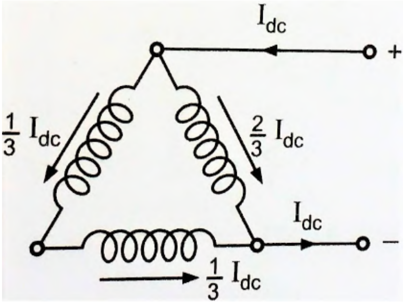

Figure 13 : Rotor circuit(c)

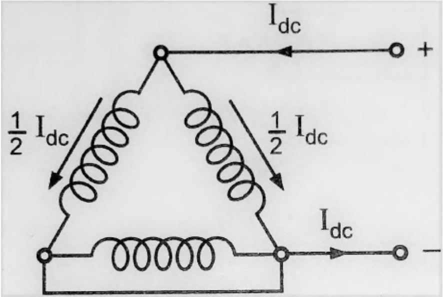

Figure 14 : Rotor circuit(d)

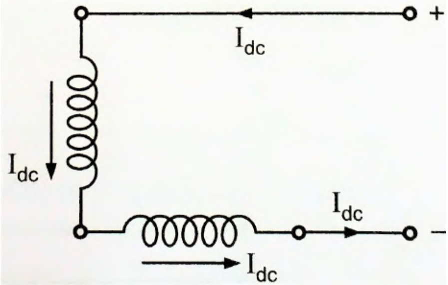

Figure 15 : Rotor circuit(e)

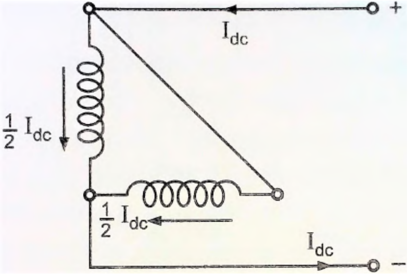

Figure 16 : Rotor circuit(f)

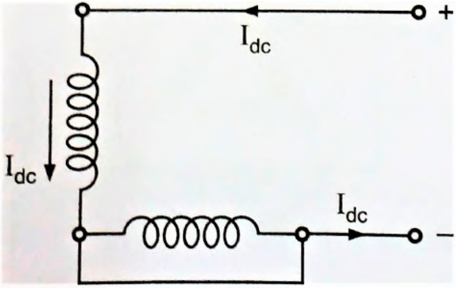

Figure 17 : Rotor circuit(g)

- **Two lead connection :** Winding is star connected & two line terminals are used for DC excitation. Third terminal is kept isolated(Fig.(a)). $\rightarrow$ non-uniform current loading of phase windings.

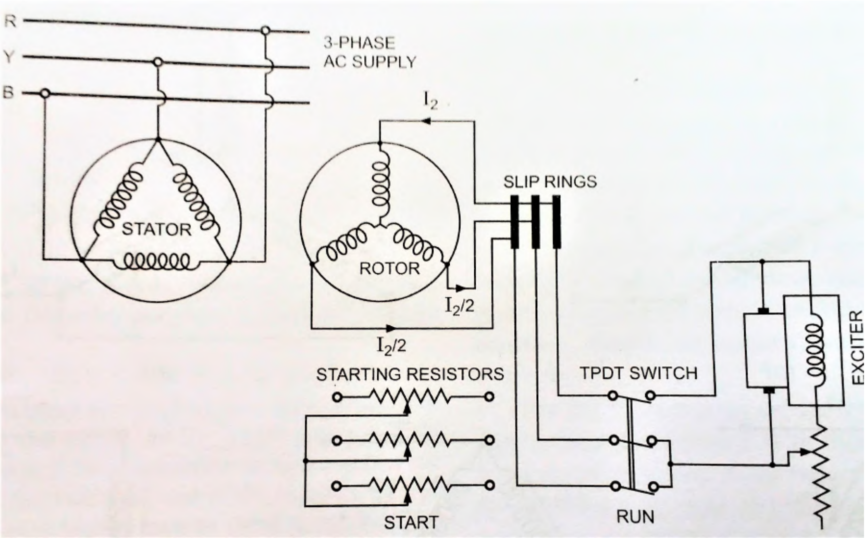

Figure 18 : Circuit for Synchronous Induction Motor

2. Synchronous Induction Motor

Starting

- Synchronous induction motor is started as induction motor by inserting resistance in the rotor circuit.
- External rotor resistance is gradually cut-out $\rightarrow$ motor attains normal induction motor speed $\rightarrow$ rotor is disconnected from starting resistance & connected to exciter $\rightarrow$ now machine will operate as synchronous motor.
- Starting resistance $\rightarrow$ low starting current $\rightarrow$ high starting torque & improved power factor.
- Synchronous induction motor is having high starting torque with low starting current(induction motor features) combined with constant speed & power factor control(synchronous motor features).
- Extra cost & low efficiency as compared to standard types.

3. Single-phase Induction Motor

- Single phase induction motors are generally used for applications which require capacity less than 0.5kW
- Merit: Simple construction, reliable, easy to repair, cheaper.
- Demerit: Low overload capacity, low efficiency, low power factor, low output as compared to 3-phase induction motor.

Construction

- Rotor is same as that of 3-phase squirrel cage induction motor.
- Single phase winding in the stator.
- Winding is usually concentric coils
- **Main winding or Running winding** and **Auxiliary winding or Starting winding**
- Centrifugal switch cut-out auxiliary(starting) winding after starting.

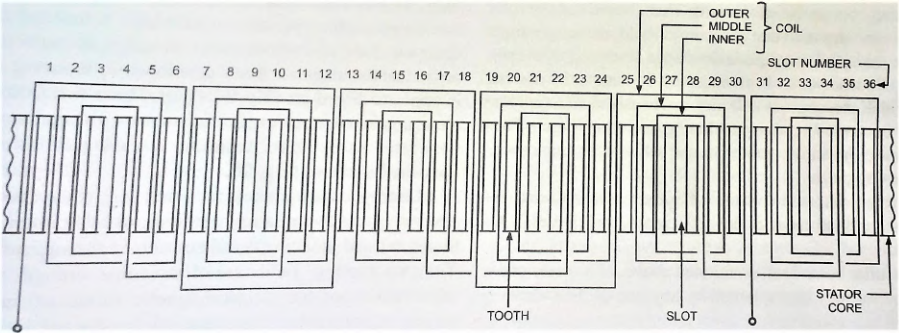

Figure 19 : Winding diagram(6-pole, 36-slots, 1-phase induction motor)

3. Single-phase Induction Motor

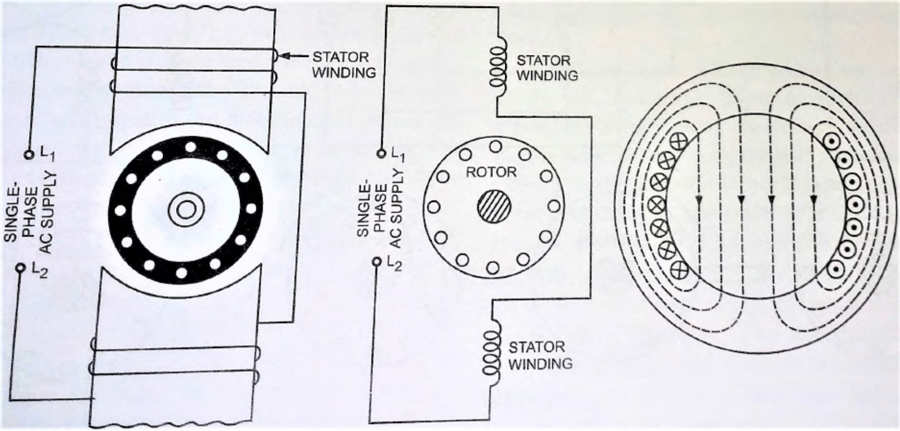

Figure 20 : Single phase induction motor

- Stator is connected to 1-phase AC supply
- Magnetic field is developed.
- Axis of magnetic field will be always along the axis of stator coils.
- Field is stationary(not rotating) but pulsates in magnitude and varies sinusoidally with time.
- Current is induced in the rotor $\rightarrow$ axis of rotor mmf coincides with that of stator field $\rightarrow$ torque angle is zero $\rightarrow$ no starting torque is developed.
- If the rotor is given a push, it will pick up speed and continue to rotate developing the operating torque.
- Single phase induction motor is **not self-starting**

- **Pulsating field produced in a single phase motor can be resolved into two components of half it's amplitude & rotating in opposite directions with synchronous speed**

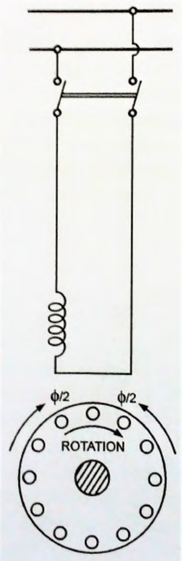

Figure 21: Circuit(a)

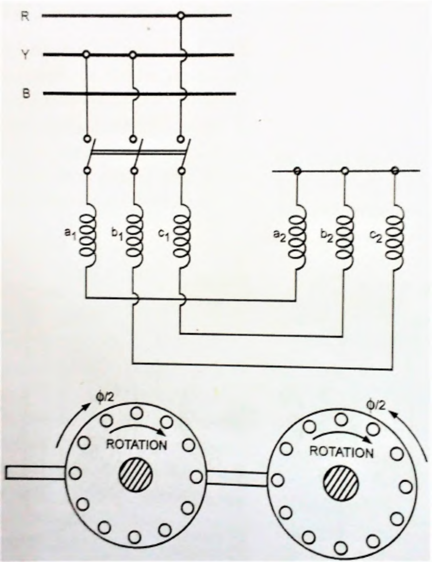

Figure 22 : Circuit(b)

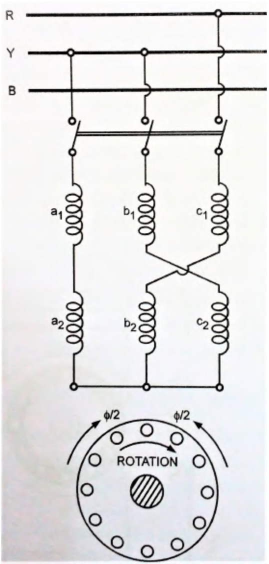

Figure 23 : Circuit(c)

- At standstill $\rightarrow$ N = 0 $\rightarrow$ Slip, s = 1 $\rightarrow$ Two rotating magnetic field slip past the rotor at same slip inducing equal currents in the squirrel cage rotor.
- Two rotating magnetic fields are of same strength $\rightarrow$ develop equal & opposite electromagnetic torques $\rightarrow$ Net torque is zero $\implies$ Starting torque is zero $\rightarrow$ Hence **single phase induction motor is not self-starting**.
- If the rotor is made to run at speed 'N' by external means in any direction(eg: in the direction of forward field) $\rightarrow$ now the slips are 's' & (2-s).
- Slip of rotor with respect to forward rotating field(F_f),

$$s_f = \frac{N_s - N}{N_s} = s$$

- Slip of rotor with respect to the backward rotating field(F_b),

$$s_b = \frac{N_s - (-N)}{N_s} = \frac{2N_s - (N_s - N)}{N_s} = (2 - s)$$

3.1 Double Revolving Field Theory

- During normal operation, (2-s) $>>$ s $\rightarrow$ backward field rotor currents are much larger than at standstill & have low power factor $\rightarrow$ corresponding opposing rotor mmf causes the backward field to be greatly reduced in strength.
- Low speed forward rotating field induces smaller currents of higher power factor in rotor than at standstill $\rightarrow$ strengthens forward field.
- Weakening of backward field & strengthening of forward field depends upon the slip(or speed of rotor). It increases with decrease in slip w.r.t forward field or increase in rotor speed in forward direction.
- **At near synchronous speed, forward field $>>$ backward field. Hence there is a net running torque.**

3.2 Torque-Slip Curve

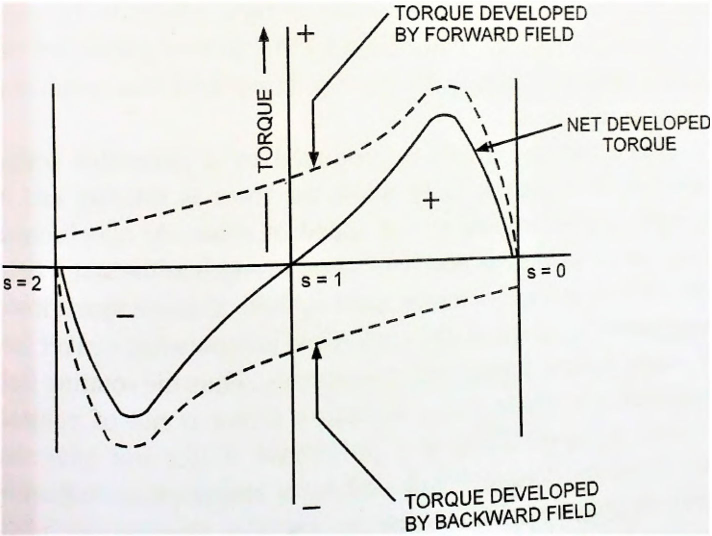

Figure 24 : Torque-Slip curve of Single Phase Induction Motor

3.2 Torque-Slip Curve

- Torque produced by forward field in the direction of rotation $\rightarrow +$ve
- Torque produced by backward field $\rightarrow$ -ve
- Net torque $=$ Algebraic sum of individual torques produced by forward & backward fields.
- **Net torque passes through zero at slip $= 1$(at standstill) $\Longrightarrow$ Starting torque $= 0 \rightarrow$ Motor can't start rotating independently.**
- For other values of slip, motor develops a net torque in the direction of rotation.
- Maximum torque is also less.
- **Resultant torque drops to zero at a speed slightly below synchronous speed**(whereas in 3-phase induction motor, torque doesn't drop to zero until synchronous speed is attained).
- Net torque at synchronous speed is negative(due to backward field).
- Single phase induction motors ar noisier than 3-phase induction motor due to the presence of second harmonic pulsating torques.

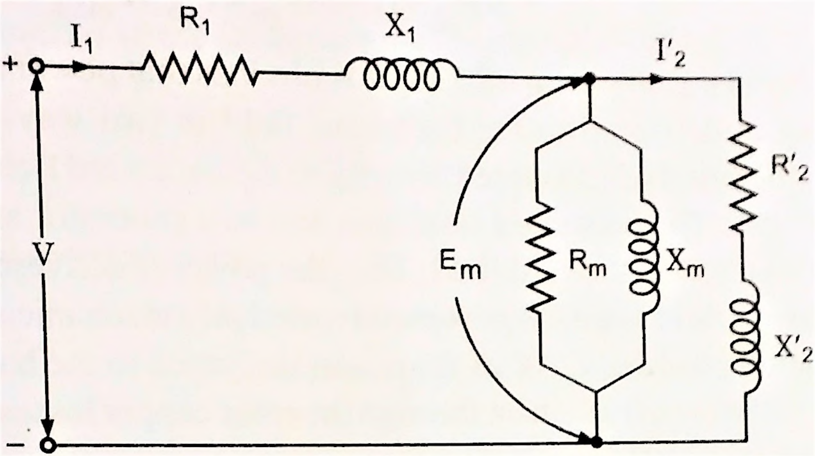

Figure 25 : Equivalent circuit at standstill

3.3 Equivalent Circuit

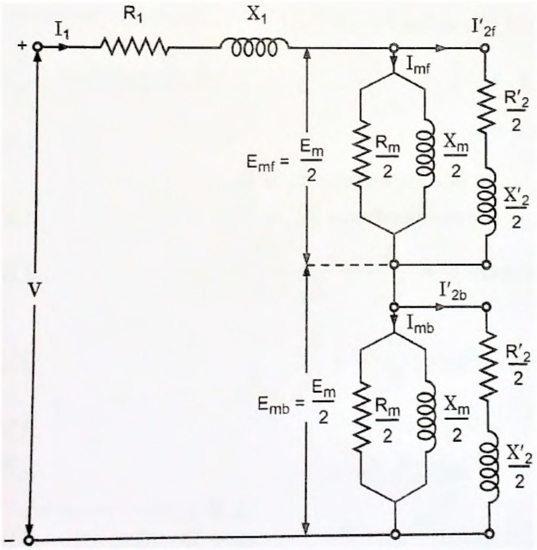

Figure 26 : Equivalent circuit at standstill based on Double Field Revolving

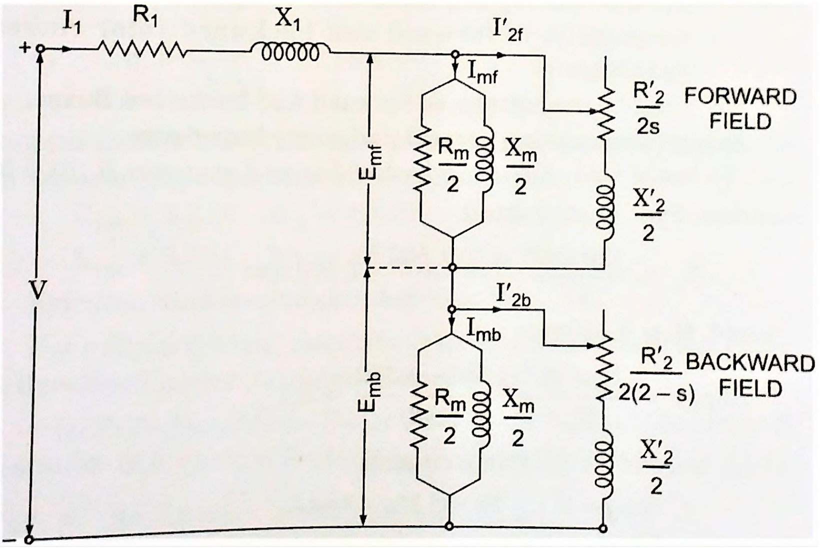

Figure 27 : Equivalent circuit under normal operation

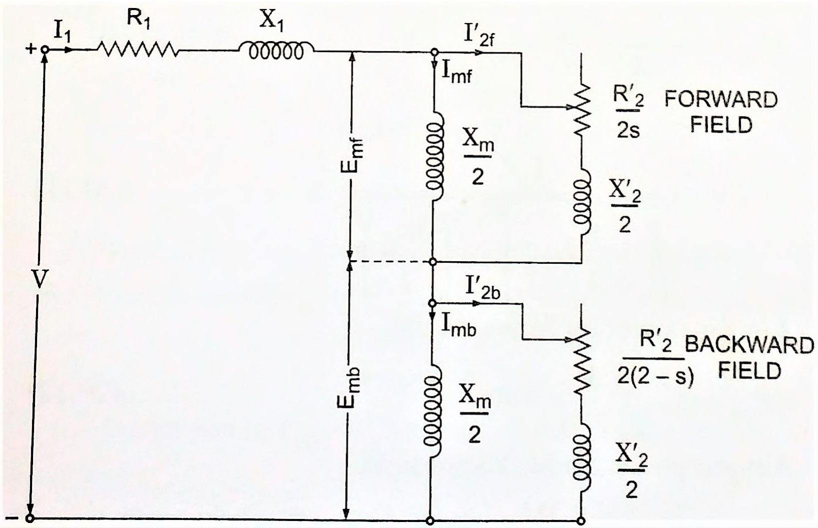

Figure 28 : Approximate equivalent circuit under normal operation

- E_m = emf induced in the stator (Fig. 25).
- Two flux components induce in the stator respective emf E_{mf} & E_{mb} (Fig. 26).
- Applied voltage(V) is the phasor sum of E_{mf}, E_{mb} & voltage drop across stator resistance(R_1) & stator leakage reactance(X_1).

$$\overrightarrow{V} = \overrightarrow{E_{mf}} + \overrightarrow{E_{mb}} + \overrightarrow{I_1}(R_1 + jX_1)$$

- When motor is running at speed 'N' w.r.t. forward field, the slip is 's' w.r.t. forward field & (2-s) w.r.t. backward rotating field (Fig. 27).
- If core losses are neglected, R_m can be eliminated $\rightarrow$ Approximate equivalent circuit (Fig. 28).

- Single phase induction motors are not self-starting. Hence it is necessary to employ some external means for making it self-starting.

Types

1. **Split-Phase Induction Motor** or **Resistance-Start Single-phase Induction Motor**
2. **Capacitor-Start Induction Motor**
3. **Capacitor-Start and Capacitor-Run Induction Motor**
4. Permanent Capacitor Single-phase Induction Motor
5. **Shaded Pole Induction Motor**

Principle of Phase Split

- If two windings, spaced 90^o electrical degrees apart & connected in parallel on the stator of a motor is excited by a single phase source, an alternating field will be produced but it will not rotate.
- If an impedance(resistance, inductance or capacitance) is connected in series with one of the windings, the currents may differ in time phase. By proper selection of impedance, the currents may be made differ by as much as 90^o $\rightarrow$ rotating field is produced(as in case of two motor) $\rightarrow$ starting torque $\rightarrow$ motor will become self-starting.
- Main winding or Running winding
- Auxiliary winding or Starting winding

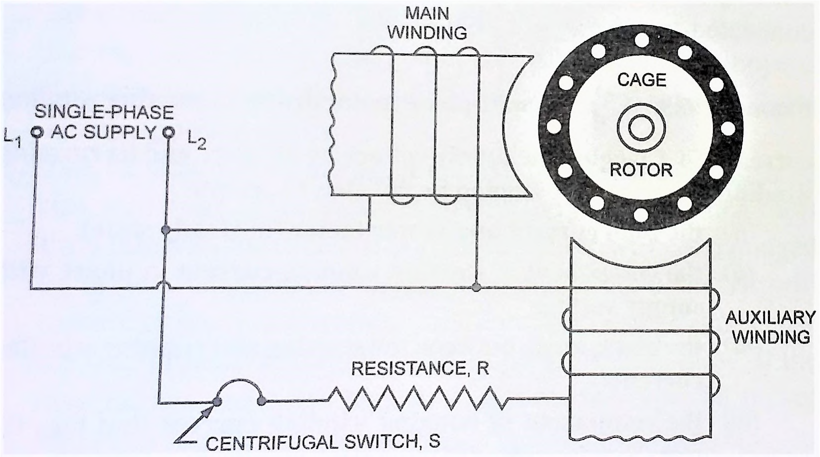

Figure 29 : Split-phase Induction Motor

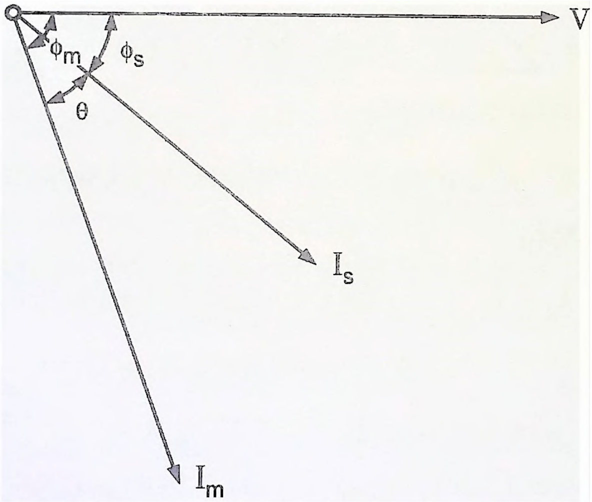

Figure 30 : Phasor Diagram

Split-Phase or Resistance-Start Induction Motor

- High resistance is connected in series with auxiliary winding.
- Main winding $\rightarrow$ High inductive reactance & Low resistance.
- Auxiliary winding $\rightarrow$ Low inductive reactance & High resistance.
- Auxiliary or starting winding current(I_s) doesn't lag much supply voltage(V).
- Main winding current(I_m) lags supply voltage(V) more than that of I_s.
- The phase shifted currents in main winding & auxiliary winding produce rotating magnetic field $\rightarrow$ motor starts.
- Auxiliary winding is removed during running condition by the centrifugal switch.
- Starting torque(T_{st}) = (1.5 to 2) times full-load torque(T_{FL})
- Starting current(I_{st}) = (6 to 8) times full-load current(I_{FL})

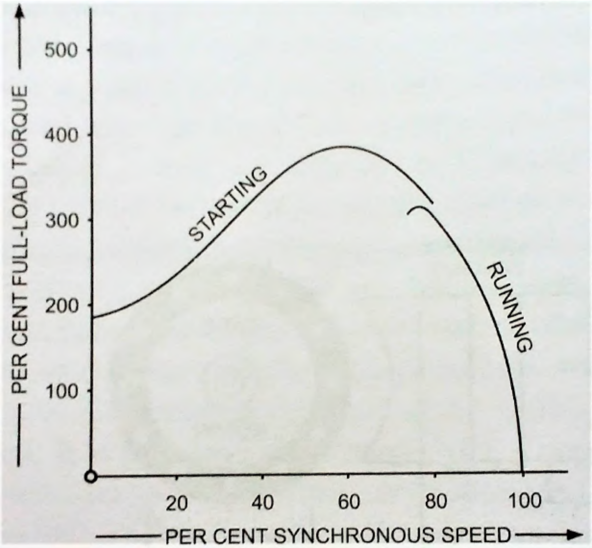

Figure 31 : Torque-Speed Curve

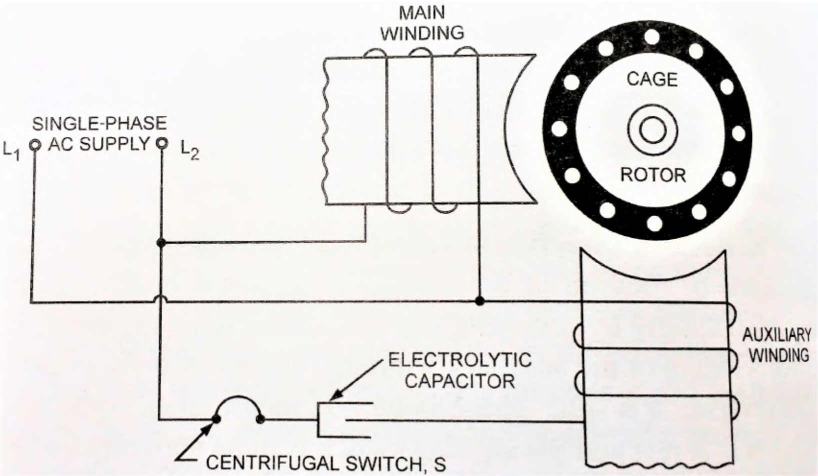

Figure 32 : Capacitor start induction motor

Capacitor-Start Induction Motor

- Also known as **Capacitor-Start Induction-Run Motor**
- An electrolytic capacitor is connected in series with the auxiliary winding.
- By using a capacitor of proper value, starting winding current(I_s) can be made to lead the main winding current(I_m) by $90^o \implies \theta = 90^o$.
- Auxiliary winding is removed during running condition by the centrifugal switch.
- Starting torque(T_{st}) = (3.5 to 4.5) times full-load torque(T_{FL})

Capacitor-Start Induction Motor

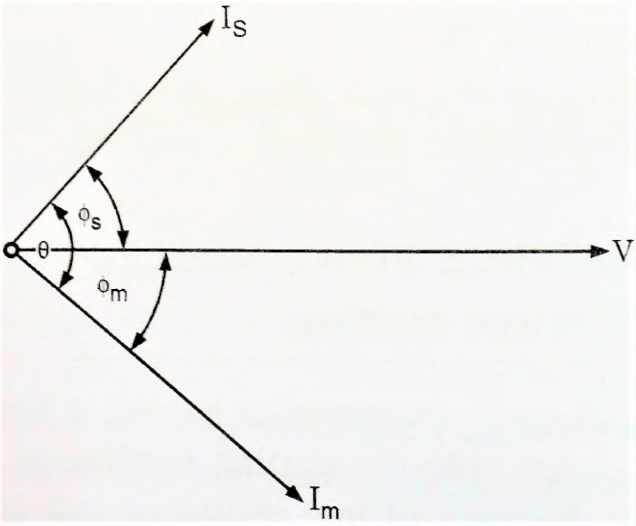

Figure 33 : Phasor diagram

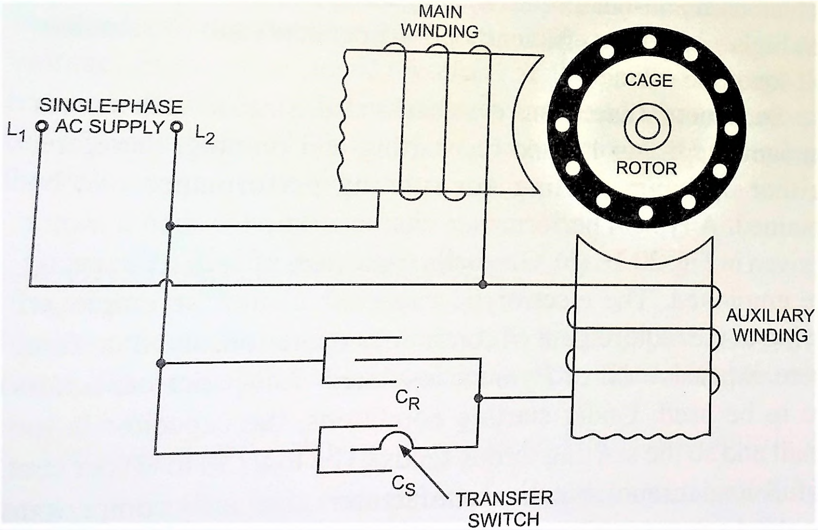

Figure 34 : Capacitor-Start and Capacitor-Run Induction Motor

- Auxiliary winding & a capacitor remain connected in the circuit at all times.
- Capacitor employed for normal running(C_R) should be of continuous-duty rating $\rightarrow$ Small value oil impregnated paper capacitor.
- Capacitor employed for starting(C_S) should be of short-duty rating $\rightarrow$ Large value electrolytic capacitor.
- When motor attains 75% of synchronous speed, starting capacitor is cut-out from the circuit by the operation of centrifugal switch.
- C_S = (10 to 15) times C_R
- Better power factor(0.8 to 0.95)
- High cost

Shaded Pole Induction Motor

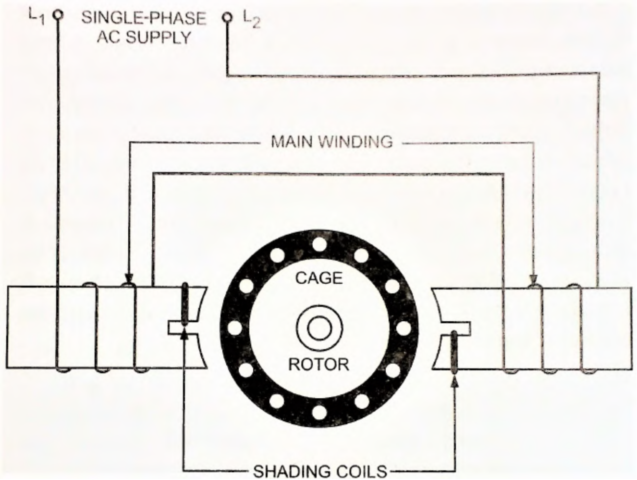

Figure 35 : Shaded Pole Induction Motor

- Stator has salient poles
- A part(about 1/4 to 1/3) of each pole is wrapped by a copper strap forming a closed loop known as **shading coil**.
- Stator winding is connected across the ac supply $\rightarrow$ alternating current flows through the winding
- Change of flux induces short-circuit currents in the shading coil in such a direction as to oppose the core flux.
- Flux in the shaded part of the pole lags behind the flux in the main part of the pole.
- $\implies$ Main flux & shaded pole flux are displaced by approximately 90^o $\rightarrow$ rotating magnetic field is set-up $\rightarrow$ torque is developed.

Working:

(i) From zero to near positive maximum current

- Current in the winding increases $\rightarrow$ emf & current are induced in shading coil $\rightarrow$ shading flux opposes main flux $\rightarrow$ greater portion of main flux passes through unshaded part of pole.
- Resultant flux lies near the centre of the unshaded part of the pole.

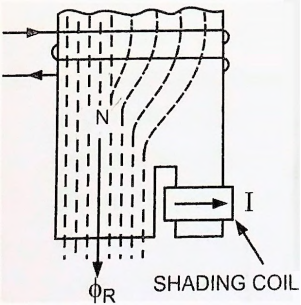

Figure 36 : Shifting of air-gap flux

(ii) In the region of maximum current

- Rate of change in current & flux are zero → induced emf & hence current in shading coil becomes zero → flux distribution is uniform over the entire pole face.
- Resultant flux lies at the centre of the pole.

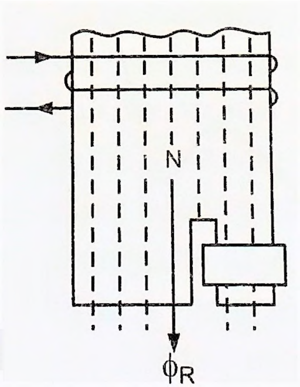

Figure 37 : Shifting of air-gap flux

(iii) From nearly positive maximum to zero

- Current in the winding decreases → emf & current are induced in shading coil → shading flux aids main flux → greater portion of main flux passes through shaded part of pole.
- Resultant flux lies at the centre of the shaded part of the pole.

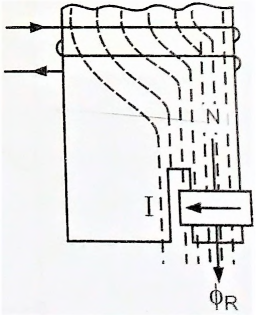

Figure 38 : Shifting of air-gap flux

→ Effect of shading coil is to cause the field flux to shift across the pole face from unshaded portion to shaded portion → Shifting flux is like a rotating weak field moving in the direction from unshaded portion to shade portion of the pole → starting torque is produced → motor starts.

3.5 Applications

Single phase induction motors are widely used in domestic as well as commercial fields.

- Washing Machines
- Fans
- Blowers
- Refrigerators
- Grinders
- Centrifugal pumps
- Hair dryers

Q1. The details of a single phase, 4-pole induction motor are follows. 230V, 50Hz, Output = 373W, Input current = 2.9A, Power factor = 0.71, Speed = 1410rpm. Calculate (i) Efficiency (ii) Slip when delivering rated output

Ans:
(i) Efficiency = 78.76%
(ii) Slip = 0.06 or 6%

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Thank You